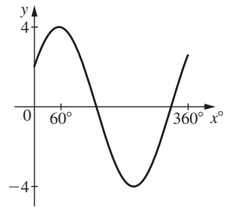
Answers to Revision Exercises

Trigonometry

1 a $\frac{7\pi}{4}$ **b** $\frac{7\pi}{6}$ **2** 7.34 cm^2 **3** 37.1m **4 a** 19.7m **b** 105° **c** 7.00mm **d** 114° **5** $\hat{B} = 88.4°$ or $\hat{B} = 18°$

6 a



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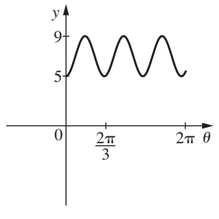

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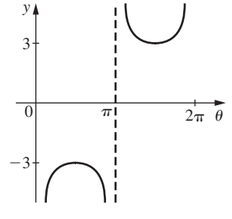
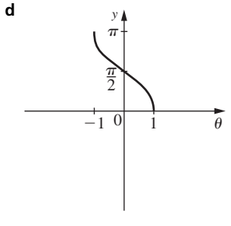
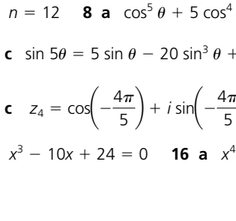


d



7 a $y = 2\sin 4\theta$ **b** $y = -3\cos 2\theta - 3$ **8 a** $\frac{7\pi}{18}, \frac{11\pi}{18}$ **b** $\frac{\pi}{3}, \frac{2\pi}{3}$ **c** $\frac{\pi}{8}, \frac{5\pi}{8}$ **9** $x = 1.08$ **10 a** 3.46m **b** 0100

10 c



12 $\theta = 0.253, 2.89, \frac{3\pi}{2}$ **13** $\frac{-\sqrt{74}}{370}$

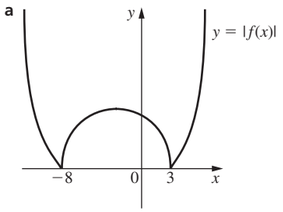
14 $\sqrt{18}\cos(2x + 70.5)°$, $x = 45°, 64.5°$

Functions

1 $-\frac{1}{2} < x < 5$ **2** $k = 2$ **3** (1,4) or $\left(-\frac{29}{8}, -\frac{5}{8}\right)$ **4** $k \leq 0$ or $k \geq 24$ **5 a** $16x^2 + 4x - 4$ **b** $4x^2 + 6x - 2$ **c** $\frac{1 + x - 4x^2}{x^2}$

6 a $\frac{5x - 9}{x - 1}$ **b** $\frac{4 - x}{3 - x}$ **c** $4x - 7$ **d** x

7 a



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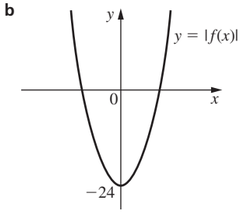


8 $x = \frac{4}{3}, x = 6$ **9** $x \geq \frac{5}{12}$

10 a $0 < y < 2$ **b** $f^{-1}(x) = \frac{1 - 3x}{x - 2}$ **11** $-7 \leq x \leq \frac{1}{3}$

12 $2x + 3$ and $x + 1$ **13** $(2x + 1)(x + 4)(x + 1)(x - 1)$

14 $x = -6, x = 5$ **15** $y = -2(x + 2)^2(x - 3)$

16 $x^3 + x^2 - 2x - 1R - 5$ **17** $a = -1, b = -4$

18 a x^6 **b** $15p^{-\frac{1}{4}}$ **c** $2 + 4x^{-2}$ **19 a** $\log_a 75$ **b** $\log_p 9$ **c** 1 **20 a** $x = 9$ **b** $x = 81$ **c** $x = 3, x = -4$ **d** $x = \frac{11}{8}$

21 a $x = 163000$ **b** $x = 1.21$ **c** $x = 1.66$ **22** $p = 2, q = 3$ **23** $x = 1$ **24** $x = y = 6$

Complex Numbers, Binomial Theorem, Sequences and Induction

1 $|z| = \sqrt{5}$, $\arg(z) = 0.464$ **2** $z = 4, z = 3 + 2i, z = 3 - 2i$ **4** $a = 4, b = 2, c = 3$ **5** $\frac{189}{32}$ **6** $a = 120, r = \frac{1}{3}$

7 $n = 12$ **8 a** $\cos^5\theta + 5\cos^4\theta i\sin\theta - 10\cos^3\theta\sin^2\theta - 10\cos^2\theta i\sin^2\theta + 5\cos\theta\sin^4\theta + i\sin^5\theta$ **b** $\cos 5\theta + i\sin 5\theta$

8 c $\sin 5\theta = 5\sin\theta - 20\sin^3\theta + 16\sin^5\theta$ $z_1 = \cos\left(\frac{2\pi}{5}\right) + i\sin\left(\frac{2\pi}{5}\right)$ $z_2 = \cos\left(\frac{4\pi}{5}\right) + i\sin\left(\frac{4\pi}{5}\right)$ $z_3 = \cos\left(-\frac{2\pi}{5}\right) + i\sin\left(-\frac{2\pi}{5}\right)$

8 c $z_4 = \cos\left(-\frac{4\pi}{5}\right) + i\sin\left(-\frac{4\pi}{5}\right)$ **9** $z_5 = 1$ **10** 40824 **12 a** $n = 10$ **b** $a = 4, r = 3$ **c** $n = 11$

15 $x^3 - 10x + 24 = 0$ **16 a** $x^4 - 12x^3 + 54x^2 - 108x + 81$ **b** $2e^{i\frac{\pi}{6}}$ **c** -64

Calculus

1 a $\frac{dy}{dx} = 3x^2 \sin 2x + 2x^3 \cos 2x$ **b** $\frac{dy}{dx} = \frac{\tan 3x}{x+1} + 3\ln(x+1)\sec^2 3x$ **c** $f'(x) = \frac{e^{2x}(2x^3 - 3x^2 - 12)}{(x^3-6)^2}$

1 d $\frac{dy}{dx} = \frac{54(5x-12)^2(3\sin x - 2)^2 \cos x - 60(3\sin x - 2)^3}{(5x-12)^3}$ **e** $\frac{dy}{dx} = \frac{-2}{3y^2\sqrt{x}}$ **f** $\frac{dy}{dx} = \cos^{-1}\sqrt{x+4} - \frac{x}{\sqrt{1-(x+4)^2}}$

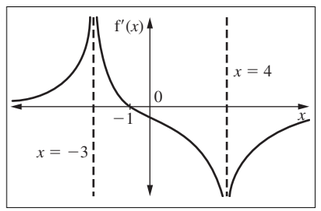
1 g $\frac{dy}{dx} = 4(x + \ln 3x)^3\left(1 + \frac{1}{x}\right)$ **h** $\frac{dy}{dx} = 4\sin x \cos x\, e^{2\sin 2x}$ **2 a** $\frac{27x^{\frac{5}{3}}}{5} - \frac{144x^{\frac{25}{12}}}{25} + \frac{8x^{\frac{5}{2}}}{5} + k$ **b** $-\frac{e^{-5x}}{125}(25x^2 + 10x + 2) + k$ **c** $\frac{6}{5}\sin^{-1}\frac{5x}{6} + k$

2 d $\frac{-4+9x}{18(4+3x)^4} + k$ **e** $\frac{x^4}{16}(4\ln 5x - 1) + k$ **f** $\frac{e^{-2x}}{10}(-\sin 4x - 2\cos 4x) + k$ **g** $x\cos^{-1}3x - \frac{\sqrt{1-9x^2}}{3} + k$

2 h $2\sin^{-1}x + 2x\sqrt{4-x^2} + k$ **3** $2y = 3x - 5$ **4 a** Area $= \frac{2}{3}$ **b** Volume $= \frac{\pi^2}{6}$ **5 a** $\frac{dy}{dx} = \frac{y-2x}{2y-x}$

5 b Parallel to x axis: $x = \pm\sqrt{3}, y = \pm 2\sqrt{3}$ Parallel to y axis: $x = \pm 2\sqrt{3}, y = \pm\sqrt{3}$

6 a



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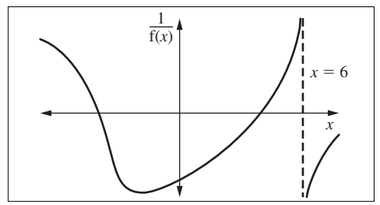
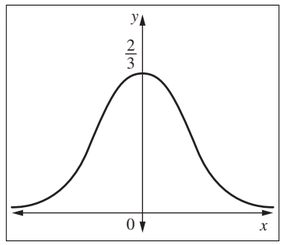


7 $6x^2 - 3$ **8 a** and **b** Minimum $(2, -3)$. Maximum $\left(-\frac{2}{3}, \frac{175}{27}\right)$ **c** $\left(\frac{2}{3}, \frac{47}{27}\right)$ **d** Concave up: $x > \frac{2}{3}$. Concave down: $x < \frac{2}{3}$.

9 $\frac{dy}{dx} = e^{\frac{x}{5}}\left(-4 + \ln\frac{\sqrt{2}}{2}\right)$ **10** Minimum: (2,7.39) **11 a** $v = \frac{\pi r^2 h}{3} + \frac{2}{3}\pi r^3$ **b** $A = \pi r\sqrt{3\left(\frac{10 - \frac{2}{3}r^3}{r^2} + r^2\right)} + \pi r^2$ **c** $r = 2.12$

12 $\frac{1}{135}\left(350\sqrt{5} - 104\sqrt{2}\right)$ **13** 1.29 **14 a** Maximum $\left(0, \frac{2}{3}\right)$ **b** $y = 0$ **c**



d 2.94

15 $A = 2, B = 1$ **16 a** $58\frac{1}{2}$m **b** 0.804 ms^{-1} **c** $423 ms^{-2}$

17 $2\pi - \frac{\pi^2}{8} - 2\pi\tan\frac{3\pi}{16}$ **18** $\ln\theta = \frac{-e^{-t}(\sin 2t + 2\cos 2t)}{5} + \ln\frac{\pi}{2} + \frac{2}{5}$

19 $r = 2\tan^{-1}t + 3 - \frac{\pi}{2}$ **20** $10240\pi^2$ **21 a** $\frac{25x}{16y}$ **b** $y = \pm\frac{5}{4}x$

Vectors and Matrices

1 $p = 4$ **2** 1.75 radians **3 a** $\mathbf{r}\cdot\left(\frac{2\mathbf{i}}{\sqrt{14}} - \frac{\mathbf{j}}{\sqrt{14}} + \frac{3\mathbf{k}}{\sqrt{14}}\right) = \frac{4}{\sqrt{14}}$ **b** 90° **c** $\mathbf{r} = \frac{35}{14}\mathbf{i} + \mathbf{j} + \lambda(-\mathbf{i} + \mathbf{j} + \mathbf{k})$ **4** $x = \frac{7}{2}, y = 14$ **5** -63

6 $q = -\frac{41}{2}$ **7 b** $p = -19$ **c** $x = \lambda, y = \frac{\lambda - 11}{11}, z = \frac{11 - 7\lambda}{11}$ **8** $p = \frac{12q - 82}{16}$ **9** Lines intersect at point with position vector $\begin{pmatrix}5\\3\\12\end{pmatrix}$

10 $x = \pm 2\sqrt{2}$ **11** $p = -\frac{13}{2}$, or $p = -1$ **12 a** $\mathbf{r} = \begin{pmatrix}1\\1\\0\end{pmatrix} + \lambda\begin{pmatrix}7\\-1\\-5\end{pmatrix}$ **13 a** $\frac{1}{k^2-k+2}\begin{pmatrix}k & 1\\-2 & k-1\end{pmatrix}$

13 b $x = \frac{3k^2-2}{k^2-k+2}, y = \frac{-k^3+k^2-6k-2}{k^2-k+2}$ **14 i** $\mathbf{r} = 3\mathbf{i} + 2\mathbf{k} + \lambda(-3\mathbf{i} - \mathbf{j} + 3\mathbf{k})$ **ii** $\sqrt{\frac{161}{2}}$ **iii** $\sqrt{322}$ **iv** $\mathbf{r}\cdot(13\mathbf{i} - 3\mathbf{j} + 12\mathbf{k}) = 63$

14 v 57.2° **vi** $\sqrt{\frac{4050}{161}}$ **15 a** $\frac{x-3}{2} = 4 - y = \frac{z+2}{3}$ **b** (7, 2, 4) **c** (11, 0, 10) **d** $4\sqrt{14}$ $x = 3 + 8\lambda$ **e** $y = -2 + 2\lambda$ $z = 3 + 7\lambda$

Probability and Statistics

1 a 1.8 **b** 4.34 **c** 2.56 **d** 1.1 **2 b** 44 **c** 63.688

2 d

Mark ≤	Cumulative Frequency
10	0
20	1
30	2
40	8
50	10
60	13
70	19
80	23
90	26
100	30
110	32

f Median = 65 Interquartile range = 43. **3 a** $\frac{1}{4}$ **b** 0.254 **c** 1.6 **d** 0.107 **e** $\sqrt[4]{8}$

4 a 480 **b** 384 **c** 480 **d** $\frac{2}{7}$ **5 a** 0.395 **b** 0.297 **c** 0.495 **d** 53 **e** 1.99 cm

5 f 3.81 cm **6 a** Median = 249 grams Interquartile range = 2 grams **b** 249.1 grams

6 c 1.74 grams **d** 1.75 grams **e** 0.303 **f** $m = 247.9$, $n = 250.2$ **g** 247.1 grams

7 a m = 1.16 **b** 0.0237 **c** 0.112 **8 a** 0.45 **b** 0.41 **c** 0.366 **9 a** 6 **b** 0.68 **c** 0.206 **d** 0.378 **e** 0.901

10 a 0.790 **b** 0.790 **c** 0.214 **11 a** 0.165 **b** 0.228 **c** 0 **d** 0.593

12 a

X	3	1	−5
$P(X = x)$	$\frac{15}{36}$	$\frac{17}{36}$	$\frac{4}{36}$

b \$1.17 **c** \$5.63 **d** 20 cents **13 a** $k = \frac{1}{384}$ **b** 0.0527 **c** 0.875 **d** 0.000244

14 a 0.676 **b** 290 **c** 20.2% **d** 0.207 **15 a** 0.00604 **b** 0.158 **c** 1 **d** 36 **e** 0.0540