

Answers

Chapter 1

Exercise 1

- 1 a 135° b 20° c 72° d 150° e 105° f 22.5° g 110° h 114.6° i 85.9° j 229.2° k 206.3° l 22.9°
2 a $\frac{\pi}{6}$ b $\frac{7\pi}{6}$ c $\frac{3\pi}{4}$ d $\frac{7\pi}{4}$ e $\frac{4\pi}{3}$ f $\frac{7\pi}{18}$ g $\frac{2\pi}{5}$ h $\frac{3\pi}{10}$ 3 a 0.611 b 1.75 c 5.24 d 1.40 e 2.30 f 4.85
4 a 30.7cm^2 b 4.71m^2 c 489cm^2 d 2430cm^2 5 a 6.28cm b 8.55m c 295cm d 113mm
6 a 66.8cm b 293cm c 177mm 7 a 126cm b 2670cm^2 8 52.2cm^2 9 46.9m^2 10 200°
11 9.30cm 12 30cm 13 99.5cm 14 7:2 15 a 17.9cm b 99.1cm^2

Chapter 1

Exercise 2

- 1 a $\frac{1}{2}$ b 0.174 c $-\frac{1}{\sqrt{2}}$ d -0.996 e $\frac{\sqrt{3}}{2}$ f $\frac{1}{\sqrt{2}}$ g $-\frac{\sqrt{3}}{2}$ h -0.737
2 a $\frac{1}{2}$ b $\frac{1}{2}$ c 1 d $\frac{\sqrt{3}}{2}$ e $\frac{1}{2}$ f $\frac{1}{\sqrt{2}}$ g $\frac{1}{\sqrt{2}}$ h 0 i -1 j 0
3 a $x^\circ = 30^\circ, 150^\circ$ b $x^\circ = 70.5^\circ, 289.5^\circ$ c $x^\circ = 41.8^\circ, 138.2^\circ$ d $x^\circ = 80.4^\circ, 279.6^\circ$ e $x^\circ = 22.0^\circ, 158.0^\circ$ f $x^\circ = 55.2^\circ, 304.8^\circ$
4 a $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$ b $\theta = \frac{\pi}{3}, \frac{2\pi}{3}$ c $\theta = \frac{\pi}{4}, \frac{3\pi}{4}$ d No Solution e $\theta = \frac{\pi}{6}, \frac{11\pi}{6}$ f $\theta = 0.290, 2.85$ g $\theta = 1.20, 5.08$ h $\theta = 0.775, 2.37$

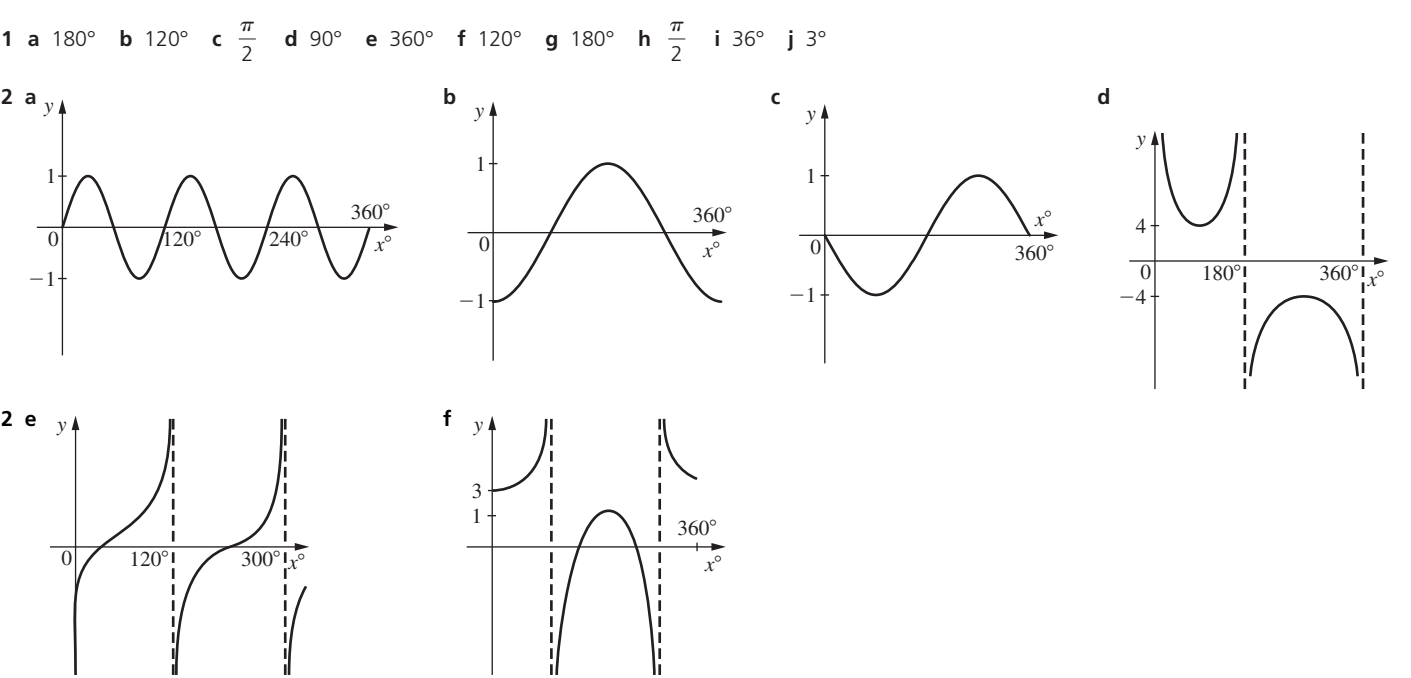
Chapter 1

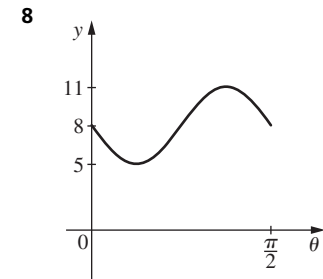
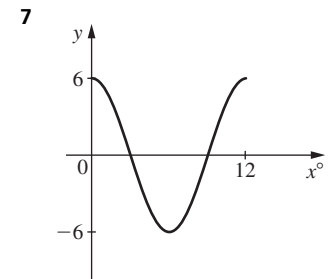
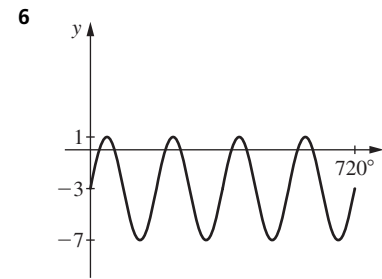
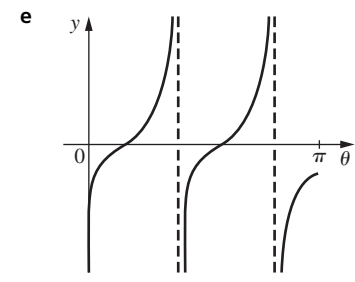
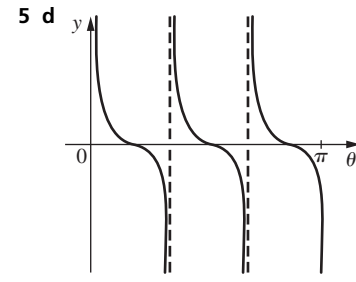
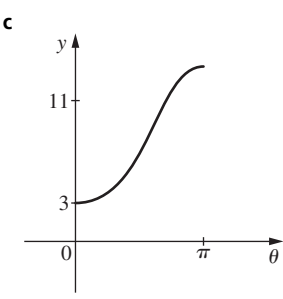
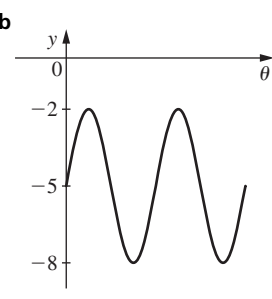
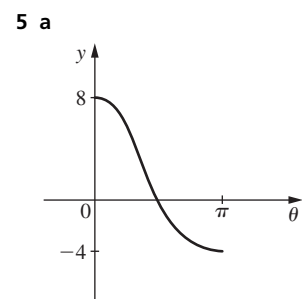
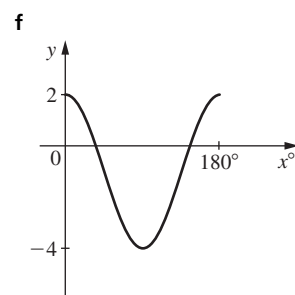
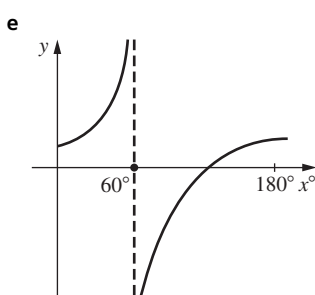
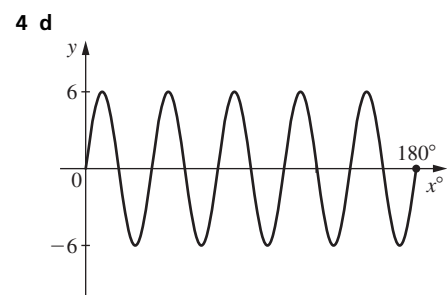
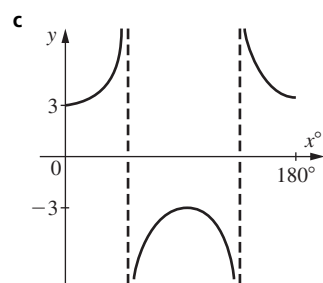
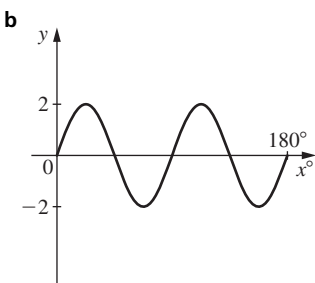
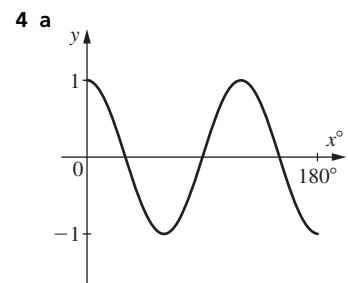
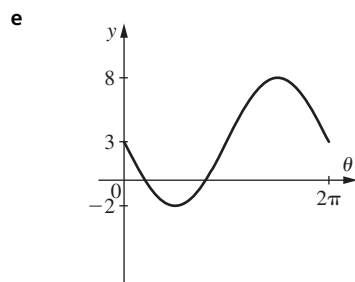
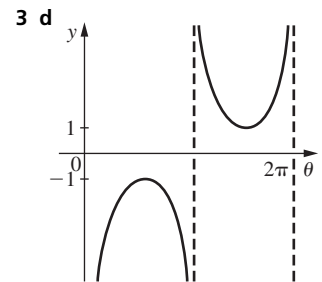
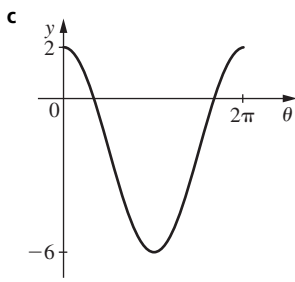
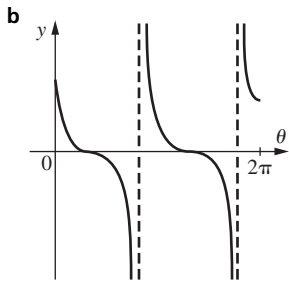
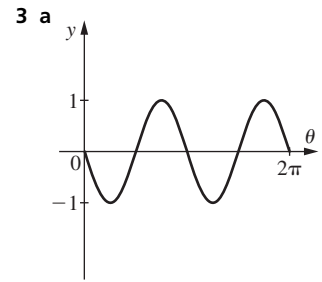
Exercise 3

- 1 a 24cm^2 b 20.7cm^2 c 160cm^2 d 26.0m^2 2 8220m^2 3 a 84.9cm^2 b 120cm^2 4 63.7m^3
5 311m^2 6 a $x = 5.82\text{cm}$ b $x = 6.37\text{cm}$ c $x = 7.64\text{m}$ d $n = 7.35\text{m}$ e $a = 14.2\text{mm}$
7 a $x^\circ = 32.4^\circ$ b $x^\circ = 127.7^\circ$ 8 44.7° or 135.3° 9 38.8° or 141.2°
10 a $x = 6.07\text{cm}$ b $x = 22.7\text{m}$ c $p = 381\text{mm}$ d $t = 10.7\text{cm}$ 11 2.63m 12 a 59.6° b 143.2° 13 68.7°
14 $A = 39.4^\circ, B = 46.5^\circ, C = 94.1^\circ$ 15 a $x = 7.64\text{m}$ b $x^\circ = 50.9^\circ$ c $x^\circ = 83.8^\circ$ d $x = 15.8\text{cm}$ e $x = 16.9\text{m}$
16 312km 17 21.6m 18 57.9° 19 56.25cm^2

Chapter 1

Exercise 4





9 a $y = 4 \cos 2\theta$ **b** $y = \sin x^\circ + 1$ **c** $y = \tan 2\theta$ **d** $y = -4 \cos x^\circ + 2$ **e** $y = -4 \sin 2\theta + 4$ **f** $y = -\frac{7}{2} \cos 4x^\circ - \frac{7}{2}$

9 g $y = \tan 15x^\circ$ **h** $y = 3 \sin 24\theta + 8$ **i** $y = 2 \csc 3x^\circ$ **j** $y = 3 \sec \frac{3}{2}\theta$

Chapter 1 Exercise 5

1 a $-\frac{1}{2}$ **b** -1 **c** $\frac{1}{2}$ **d** $\frac{1}{2}$ **e** 1 **f** $-\frac{\sqrt{3}}{2}$ **g** $-\sqrt{3}$ **h** $-\frac{\sqrt{3}}{2}$ **i** $\frac{\sqrt{3}}{2}$ **j** $-\frac{\sqrt{3}}{2}$

2 a $\frac{1}{\sqrt{3}}$ **b** $\frac{1}{\sqrt{2}}$ **c** $\frac{\sqrt{3}}{2}$ **d** $-\sqrt{3}$ **e** $-\frac{1}{\sqrt{2}}$ **f** $-\frac{1}{\sqrt{3}}$ **g** 0 **h** $-\frac{\sqrt{3}}{2}$ **i** 1 **j** $4\sqrt{3}$

3 a $\sin 43^\circ$ **b** $\cos 50^\circ$ **c** $\tan 20^\circ$ **d** $-\sin 50^\circ$ **e** $-\cos 23^\circ$ **f** $-\tan 34^\circ$ **g** $-\cos 15^\circ$ **h** $-\sin 20^\circ$ **i** $-\tan 46^\circ$

4 a $x^\circ = 30^\circ, 150^\circ$ **b** $x^\circ = 30^\circ, 330^\circ$ **c** $x^\circ = 60^\circ, 240^\circ$ **d** $x^\circ = 135^\circ, 315^\circ$

5 a $\theta = \frac{\pi}{3}, \frac{2\pi}{3}$ **b** $\theta = \frac{\pi}{6}, \frac{7\pi}{6}$ **c** $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$ **d** $\theta = \frac{5\pi}{6}, \frac{7\pi}{6}$

Chapter 1 Exercise 6

1 a $60^\circ, 240^\circ$ **b** $60^\circ, 300^\circ$ **c** $60^\circ, 120^\circ$ **d** $210^\circ, 330^\circ$ **e** $150^\circ, 210^\circ$ **f** 180° **g** 90° **h** $30^\circ, 150^\circ$ **i** $45^\circ, 225^\circ$

2 a $\frac{\pi}{6}, \frac{11\pi}{6}$ **b** $\frac{7\pi}{6}, \frac{11\pi}{6}$ **c** $\frac{5\pi}{6}, \frac{11\pi}{6}$ **d** $\frac{\pi}{4}, \frac{5\pi}{4}$ **e** $\frac{\pi}{6}, \frac{5\pi}{6}$ **f** $\frac{\pi}{6}, \frac{7\pi}{6}$ **g** $\frac{\pi}{2}, \frac{5\pi}{6}$ **h** $\frac{\pi}{6}, \frac{11\pi}{6}$

3 a $15^\circ, 75^\circ, 195^\circ, 255^\circ$ **b** $10^\circ, 110^\circ, 130^\circ, 230^\circ, 250^\circ, 350^\circ$ **c** $11.25^\circ, 56.25^\circ, 101.25^\circ, 146.25^\circ, 191.25^\circ, 236.25^\circ, 281.25^\circ, 326.25^\circ$

3 d $60^\circ, 120^\circ, 240^\circ, 300^\circ$ **e** $25^\circ, 45^\circ, 145^\circ, 165^\circ, 265^\circ, 285^\circ$ **f** $40^\circ, 80^\circ, 160^\circ, 200^\circ, 280^\circ, 320^\circ$

4 a $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$ **b** $\frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}$ **c** $\frac{\pi}{30}, \frac{5\pi}{30}, \frac{13\pi}{30}, \frac{17\pi}{30}, \frac{25\pi}{30}, \frac{29\pi}{30}, \frac{37\pi}{30}, \frac{41\pi}{30}, \frac{49\pi}{30}, \frac{53\pi}{30}$

4 d $\frac{5\pi}{12}, \frac{7\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}$ **5** $15^\circ, 105^\circ$ **6** $\frac{7\pi}{24}, \frac{11\pi}{24}, \frac{19\pi}{24}, \frac{23\pi}{24}$ **7** $1^\circ, 5^\circ, 13^\circ, 17^\circ$ **8** $-120^\circ, 60^\circ$ **9** $-\frac{8\pi}{9}, -\frac{4\pi}{9}, -\frac{2\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9}$

10 a $19.5^\circ, 160.5^\circ$ **b** $41.4^\circ, 318.6^\circ$ **c** $58.0^\circ, 238^\circ$ **d** $48.2^\circ, 311.8^\circ$ **e** $48.6^\circ, 131.4^\circ$

10 f $31.3^\circ, 288.7^\circ$ **g** $75.5^\circ, 284.5^\circ$ **h** $45^\circ, 135^\circ$ **i** $228.7^\circ, 341.3^\circ$ **j** $36.9^\circ, 143.1^\circ$

10 k $20.9^\circ, 159.1^\circ, 200.9^\circ, 339.1^\circ$ **l** $25.2^\circ, 94.8^\circ, 145.2^\circ, 214.8^\circ, 265.2^\circ, 334.8^\circ$

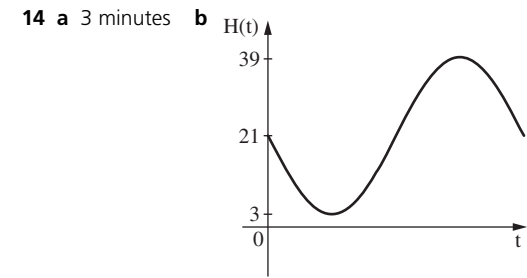
10 m $11.1^\circ, 47.1^\circ, 83.1^\circ, 119.1^\circ, 155.1^\circ, 191.1^\circ, 227.1^\circ, 263.1^\circ, 299.1^\circ, 335.1^\circ$ **n** $70.5^\circ, 289.5^\circ$

10 o $8.2^\circ, 171.8^\circ$ **p** $48.2^\circ, 311.8^\circ$ **q** $33.7^\circ, 213.7^\circ$ **r** $15^\circ, 75^\circ, 105^\circ, 165^\circ, 195^\circ, 255^\circ, 285^\circ, 345^\circ$ **s** $1.4^\circ, 4.6^\circ, 13.4^\circ, 16.6^\circ, \dots$

11 a $0.253, 2.89$ **b** $2.03, 4.25$ **c** $1.17, 4.31$ **d** $1.11, 5.18$ **e** $2.01, 0.08$ **f** $3.34, 6.08$ **g** $1.23, 4.37$ **h** $0.41, 1.68, 2.50, 3.78, 4.60, 5.87$

11 i $0.08, 1.49, 3.22, 4.63$ **j** $0.47, 1.26, 2.04, 2.83, 3.61, 4.40, 5.18, 5.97$ **k** $0.28, 0.76, 2.38, 2.86, 4.47, 4.95$ **l** $1.14, 5.15$ **m** $3.45, 5.98$

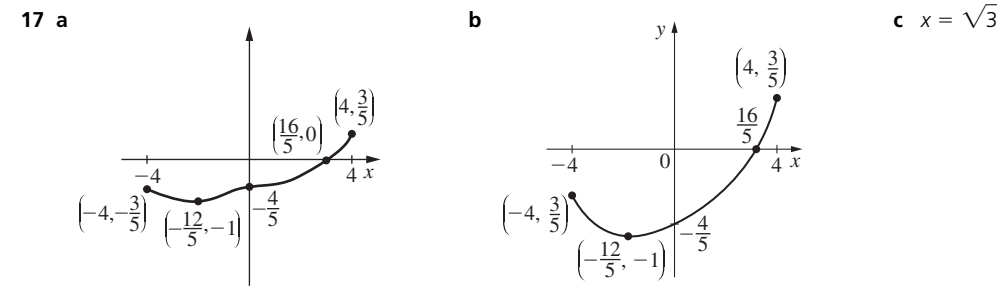
11 n $0.14, 3.28$ **o** $0.11, 0.67, 1.68, 2.25, 3.25, 3.82, 4.82, 5.39$ **12** $70.5^\circ, 289.5^\circ, 430.5^\circ, 649.5^\circ$ **13** $-\frac{35\pi}{36}, -\frac{19\pi}{36}, -\frac{11\pi}{36}, \frac{5\pi}{36}, \frac{13\pi}{36}, \frac{29\pi}{36}$



14 c i 2 mins 15 secs **ii** 45 secs **15 a** 7500 **b** 7060 **c** 12 years, 4500 fish

Chapter 1 Review Exercise

- 1 a** 30° **b** 75° **2 a** $\frac{2\pi}{3}$ **b** $\frac{13\pi}{12}$ **3** 7.01 cm^2 **4** 14.0 m **5** 20.6 cm^2
- 6 a** 15.1 cm **b** 44.8° **c** 48.9° **d** 8.63 mm **7** $2.35 < BC < 5$
- 8 a** $-\frac{1}{2}$ **b** $\frac{1}{\sqrt{2}}$ **c** $-\frac{1}{\sqrt{3}}$ **d** $-\frac{1}{2}$ **e** $\frac{1}{\sqrt{2}}$ **f** $-\frac{\sqrt{3}}{2}$ **g** $\sqrt{3}$ **h** $-\frac{1}{\sqrt{2}}$ **i** $-\frac{1}{\sqrt{3}}$ **j** 2 **k** $-\frac{2}{\sqrt{3}}$
- 9 a**
-
- b**
-
- c**
-
- 9 d**
-
- e**
-
- 10 a** $y = -3 \cos 2\theta - 1$ **b** $y = \frac{5}{2} \cos(x - 20)^\circ + \frac{5}{2}$ **11 a** $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ **b** $\theta = \frac{5\pi}{6}, \frac{7\pi}{6}$ **c** $\theta = \frac{\pi}{4}, \frac{5\pi}{4}$
- 11 d** $\theta = \frac{\pi}{12}, \frac{\pi}{6}, \frac{7\pi}{12}, \frac{2\pi}{3}, \frac{13\pi}{12}, \frac{7\pi}{6}, \frac{19\pi}{12}, \frac{5\pi}{3}$ **e** $\theta = \frac{5\pi}{12}, \frac{11\pi}{12}, \frac{17\pi}{12}, \frac{23\pi}{12}$ **12 a** $x^\circ = 135^\circ, 315^\circ$ **b** $x^\circ = 90^\circ$ **c** $x^\circ = 210^\circ, 330^\circ$
- 12 d** $x^\circ = 60^\circ, 240^\circ$ **e** $x^\circ = 15^\circ, 165^\circ, 195^\circ, 345^\circ$ **f** $x^\circ = 10^\circ, 50^\circ, 130^\circ, 170^\circ, 250^\circ, 290^\circ$
- 13 a** $x^\circ = 64.6^\circ, 295.4^\circ$ **b** $x^\circ = 109.3^\circ, 160.7^\circ, 289.3^\circ, 340.7^\circ$ **c** $20.7^\circ, 80.7^\circ, 140.7^\circ, 200.7^\circ, 260.7^\circ, 320.7^\circ$ **d** $64.6^\circ, 295.4^\circ$
- 14** $-\frac{\pi}{3}, 0, \frac{\pi}{3}$ **15** $20.9^\circ, 69.1^\circ$ **16 a** 14.5 cm **b** 0.169 s



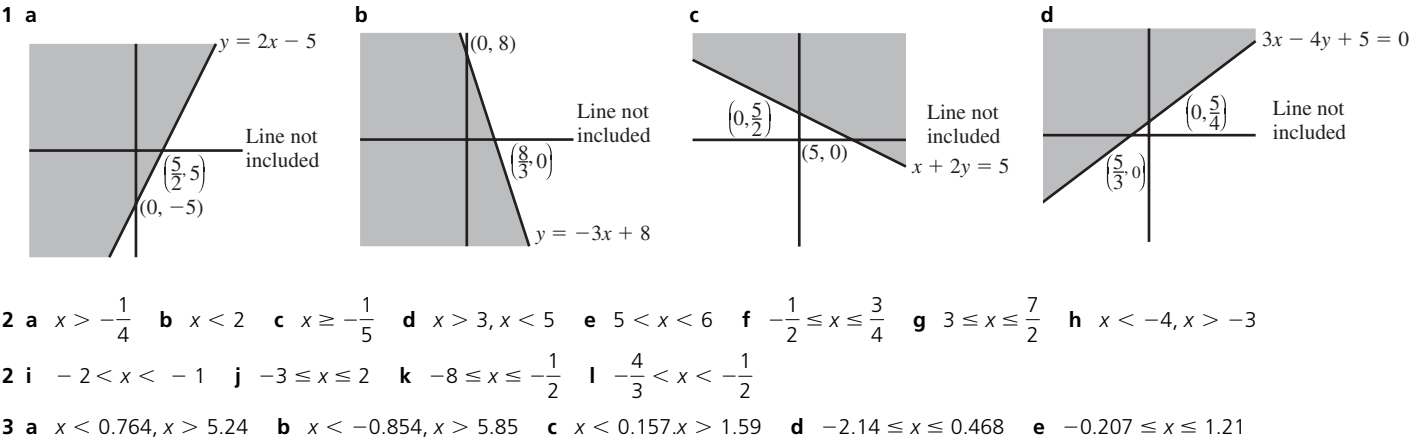
Chapter 2 Exercise 1

- 1 a** $x = 4, x = 1$ **b** $x = -3, x = 2$ **c** $x = -\frac{1}{2}, x = -8$ **d** $x = -1, x = 3$
- 2 a** $x = 2, x = 5$ **b** $x = -3, x = 8$ **c** $x = -4, x = \frac{3}{2}$ **d** $x = -\frac{3}{2}, x = \frac{2}{3}$ **e** $x = \frac{2}{3}, x = 3$
- 3 a** $x = 4.73, x = 1.27$ **b** $x = -0.854, x = 5.85$ **c** $x = -0.314, x = -3.19$ **d** $x = -0.260, x = -1.54$ **e** $x = -2.14, x = 0.468$
- 4 a** $x = -5.45, x = -0.551$ **b** $x = -1.61, x = 5.61$ **c** $x = -1.77, -0.566$ **d** $x = -3.91, x = 1.41$ **e** $x = -1.78, x = 0.281$

Chapter 2 Exercise 2

- 1 a** $(x + 1)^2 + 4$ **b** $\left(x - \frac{3}{2}\right)^2 + \frac{3}{4}$ **c** $-\left(x - \frac{3}{2}\right)^2 - \frac{11}{4}$ **d** $3(x + 1)^2 - 11$ **e** $5\left(x + \frac{7}{10}\right)^2 - \frac{109}{20}$
- 2 a** $(x + 3)^2 - 5$ Minimum $(-3, -5)$ y intercept: 4 x intercepts: $-0.764, -5.24$
- 2 b** $(x - 2)^2 - 1$ Minimum $(2, -1)$ y intercept: 3 x intercepts: 3, 1
- 2 c** $\left(x + \frac{5}{2}\right)^2 - \frac{17}{4}$ Minimum $\left(-\frac{5}{2}, -\frac{17}{4}\right)$ y intercept: 2 x intercepts: 4.56, 0.438
- 2 d** $-(x + 2)^2 + 7$ Maximum $(-2, 7)$ y intercept: 3 x intercepts: $-4.65, 0.646$
- 2 e** $-(x - 4)^2 + 19$ Maximum $(4, -19)$ y intercept: 3 x intercepts: $-0.359, 8.36$
- 2 f** $2\left(x + \frac{5}{2}\right)^2 - \frac{47}{2}$ Minimum $\left(-\frac{5}{2}, -\frac{47}{2}\right)$ y intercept: -11 x intercepts: 0.928, -5.93
- 2 g** $4\left(x - \frac{3}{8}\right)^2 + \frac{7}{16}$ Minimum $\left(\frac{3}{8}, \frac{7}{16}\right)$ y intercept: 1 x intercepts: none
- 2 h** $3\left(x - \frac{5}{6}\right)^2 - \frac{1}{12}$ Minimum $\left(\frac{5}{6}, -\frac{1}{12}\right)$ y intercept: 2 x intercepts: $-1, -\frac{2}{3}$
- 2 i** $-2\left(x - \frac{3}{4}\right)^2 + \frac{23}{8}$ Minimum $\left(\frac{3}{4}, \frac{23}{8}\right)$ y intercept: 4 x intercepts: $-0.851, 2.35$
- 3 a** Minimum $(-1.25, -10.125)$ y intercept: -7 x intercepts: $-3.5, 1$
- 3 b** Maximum $(2.5, -0.75)$ y intercept: -7 x intercepts: none **c** Minimum $(-0.6, 14.2)$ y intercept: 16 x intercepts: none
- 3 d** Maximum $(0.833, 11.1)$ y intercept: 9 x intercepts: $-1.09, 2.76$

Chapter 2 Exercise 3



Chapter 2 Exercise 4

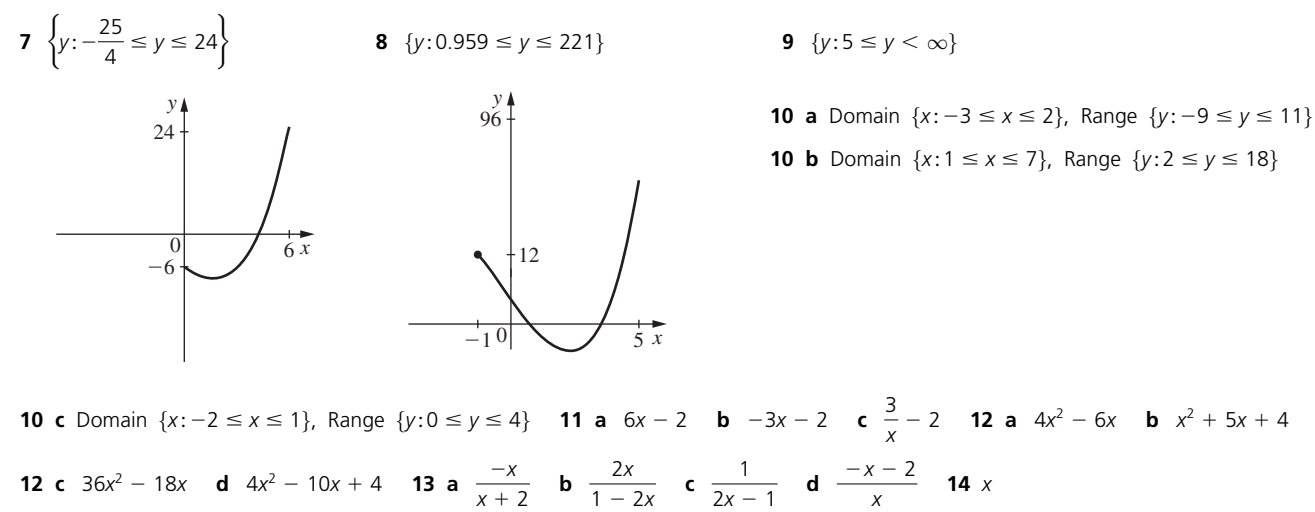
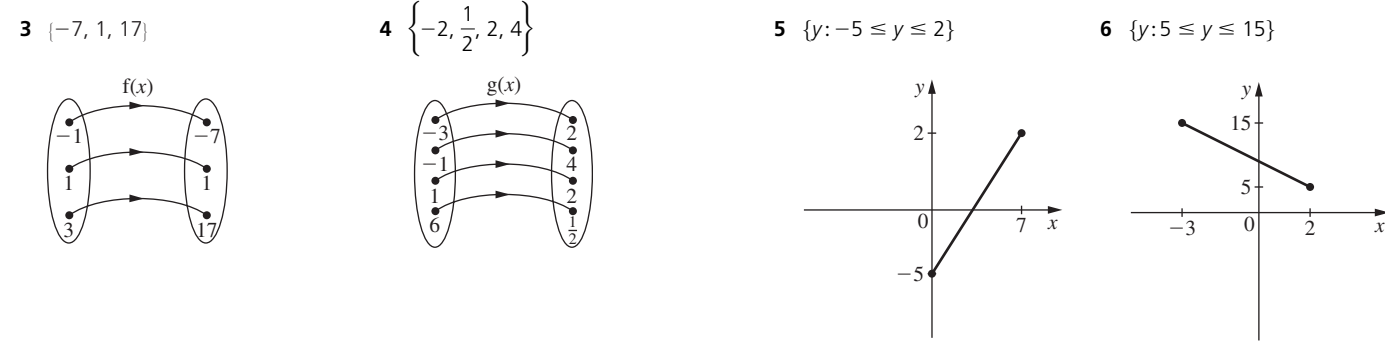
- 1 a** Imaginary roots **b** Real distinct roots **c** Imaginary roots **d** Real distinct roots **e** Real equal roots **f** Imaginary roots
- 2** $p = \pm 14$ **3** $q = \frac{9}{8}$ **5** $a - b - 8ab^3 + 16a^2b = 0$ **7** $p = \frac{1}{2}$ **9** $m = \pm\sqrt{\frac{32}{9}}$ **11 a** (ii) **b** (iii) **c** (i) **d** (iii)

Chapter 2 Review Exercise

- 1 $4 - (x - 2)^2$ 2 $-1 < x < 3$ 3 $k = 4.08, k = -2.58$ 4 i. $x = \frac{1}{3}, -2, y = \frac{7}{3}, 0$ ii $x = \frac{9}{2}, 6, y = \frac{9}{2}, 3$ 5 $-2.46 < k < 0.458$
- 6 $m = \pm 2$ 8 $x = 16, x = 1$ 9 $k \leq 0, k \geq 8$ 10 $a = \frac{9}{4}, b = \frac{7}{2}$. Maximum point is $(\frac{7}{2}, \frac{9}{4})$. Line of symmetry is $x = \frac{7}{2}$
- 11 6 years old 12 $\cos C = \frac{c^2 + 3}{4c}$. True for all real values of c .

Chapter 3 Exercise 1

- 1 a 11 b 1 c 130 d 6 2 a -17 b 12 c -27 d $\frac{19}{4}$



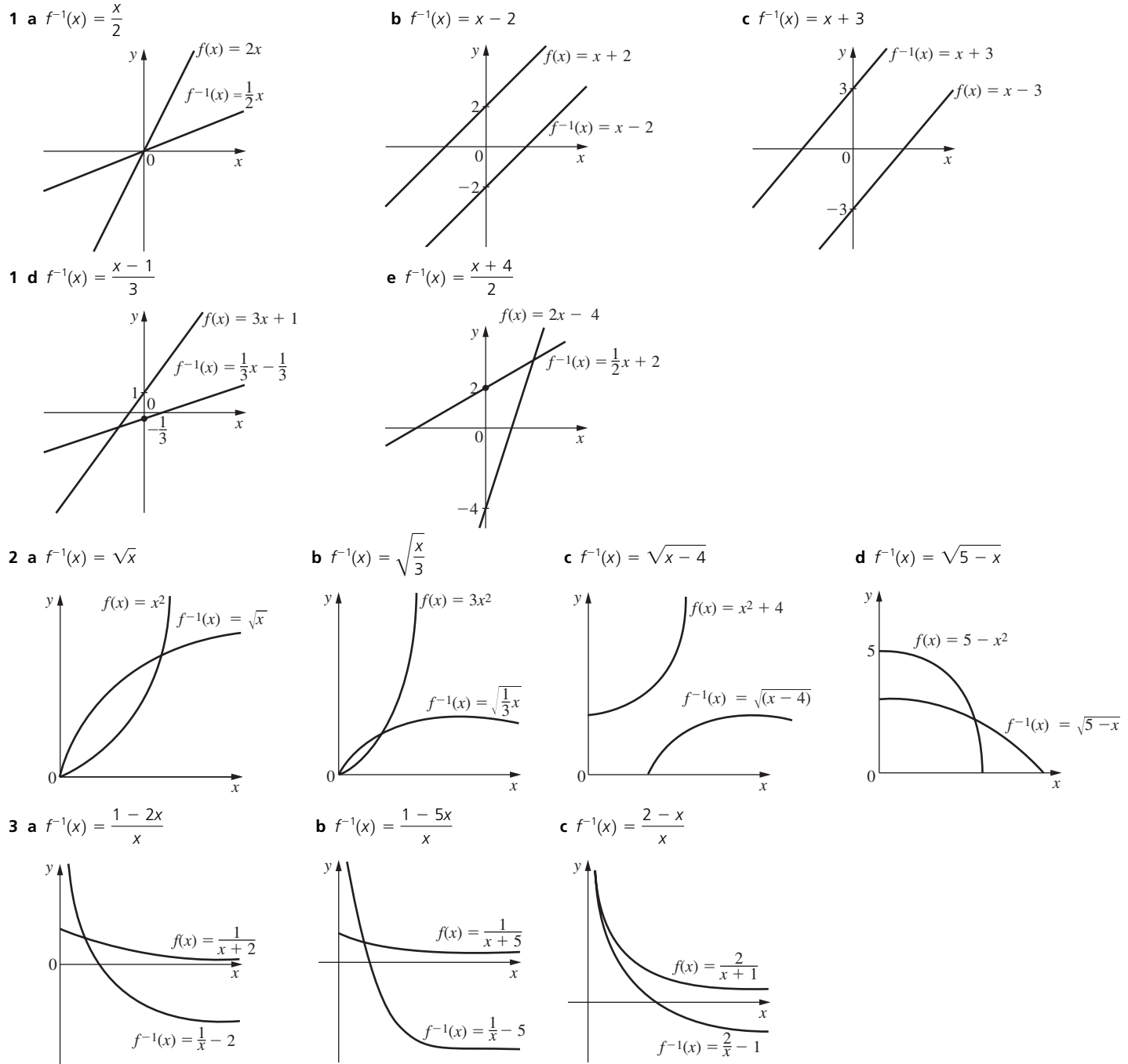
Chapter 3 Exercise 2

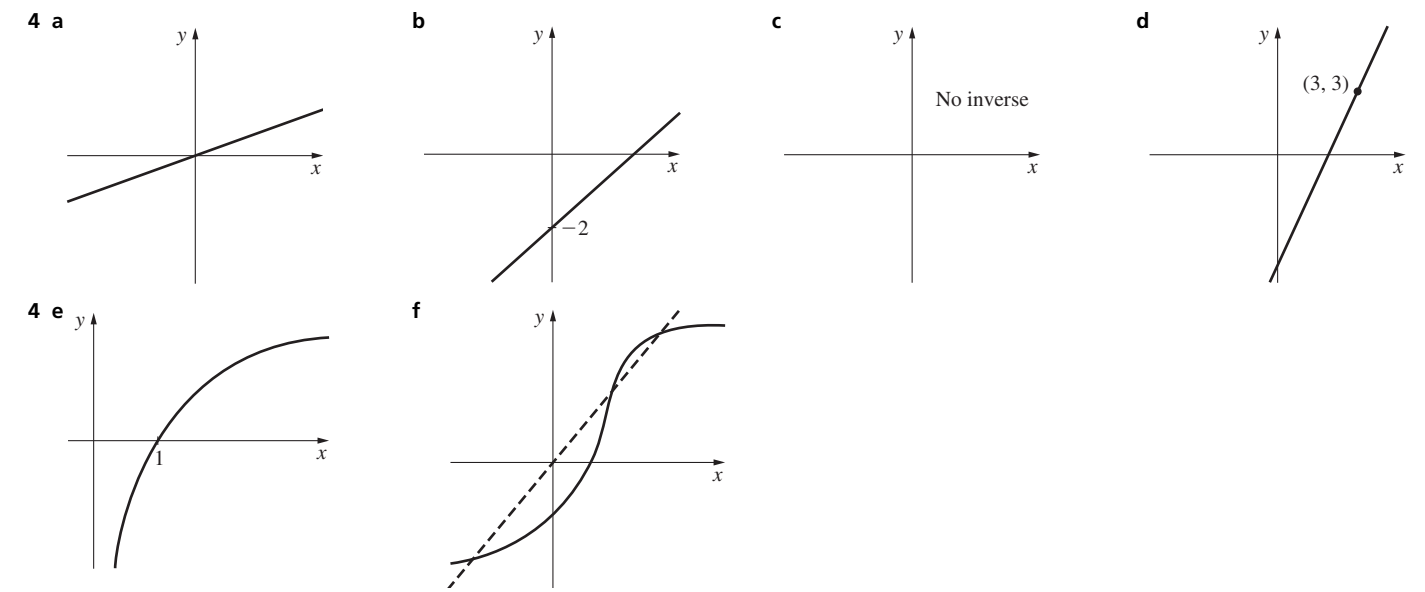
- 1 a 14 b 24 c 2 d $4x + 24$ 2 a 61 b 5 c 46 d $3x^2 - 7$ 3 a $\frac{\pi + 3}{3}$ b $\frac{\sqrt{3}}{2}$ c $\frac{\sqrt{3}}{2}$ d $\sin(\theta + \frac{\pi}{3})$
- 4 a i $2x^2 - 1$ ii $(2x - 1)^2$ b i $9x^2 + 30x + 21$ ii $3x^2 - 7$ c i $(x^2 - 6)^3$ ii $x^6 - 6$ d i $\cos(3x^2)$ ii $3 \cos^2 x$
- 4 e i $8x^3 + 36x^2 + 52x + 31$ ii $2x^3 - 2x + 17$ f i $(x^2 + 4)^6 + 2(x^2 + 4) + 3$ ii $(x^6 - 2x + 3)^2 + 4$ g i $\sin(2x^2 - 14)$ ii $\sin^2 2x - 7$
- 5 a $6x - 3p + 4$ b $6x + 8 - p$ c $p = -2$ 6 a $12x^2 - 3$ b $6(2x - 3)^2$ c $216x^4$ d $4x - 9$ 7 a $\cos(x + \frac{\pi}{2})$ b $\cos x + \frac{\pi}{2}$
- 7 c $x + \pi$ d $\cos(\cos(x))$ 8 a i $\frac{2}{3x-2}$ ii $\frac{x+3}{x-3}$ b i $\frac{9-9x}{x^2}$ ii $\frac{3}{x^2-3x}$ c i $\frac{2-3x}{x+1}$ ii $\frac{2-5x}{3-5x}$
- 8 d i $\frac{2x}{3-x}$ ii $\frac{3x-1}{2}$ e i $\frac{1}{x+1}$ ii $\frac{2x+4}{x}$ 9 x

Chapter 3 Exercise 3

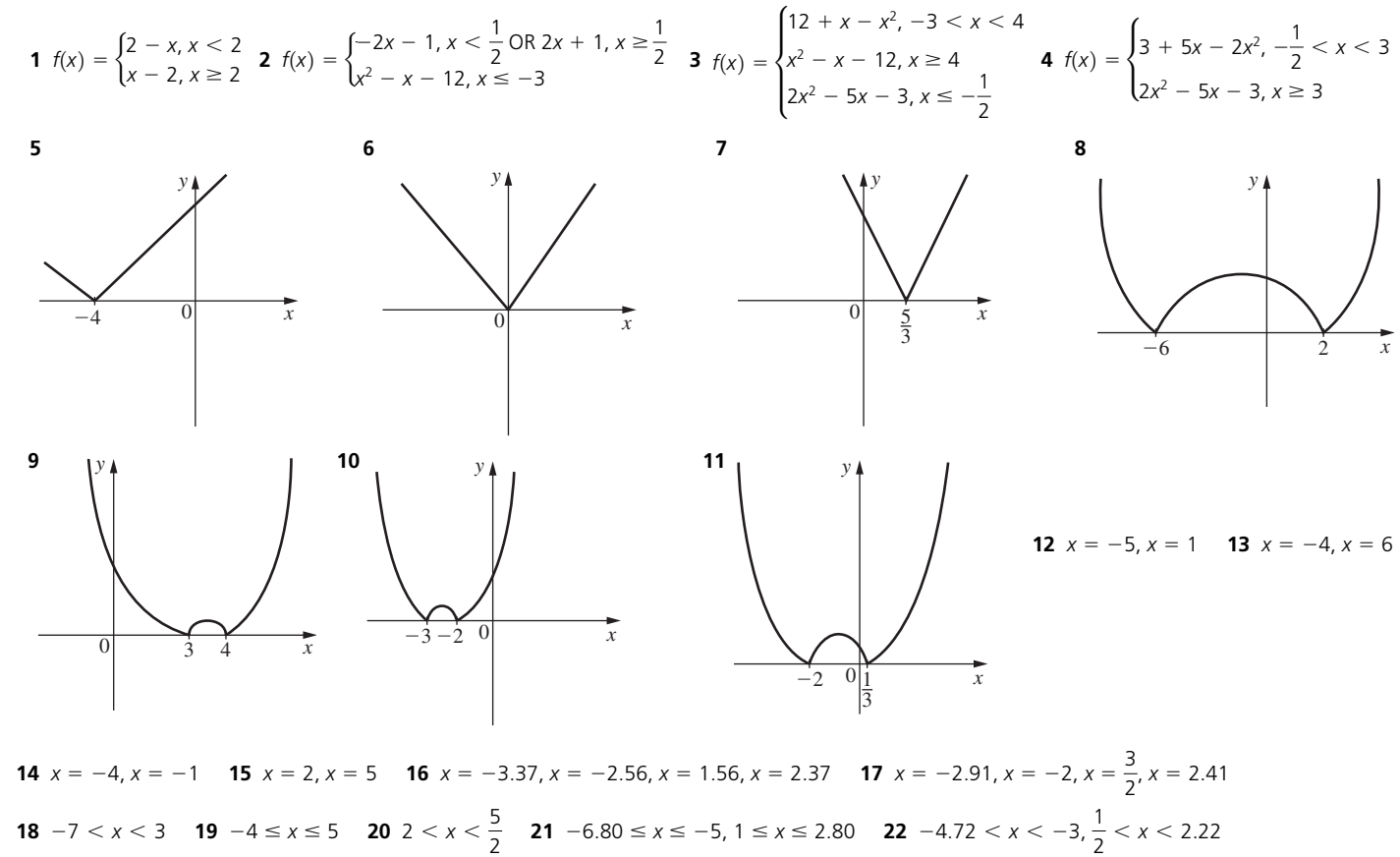
- 1 a yes b no c yes d no 2 a $f^{-1}(x) = \frac{x}{4}$ b $f^{-1}(x) = x + 5$ c $f^{-1}(x) = x - 6$ d $f^{-1}(x) = \frac{3}{2}x$ e $f^{-1}(x) = 7 - x$
- 2 f $f^{-1}(x) = \frac{9-x}{4}$ g $f^{-1}(x) = \frac{x-9}{2}$ h $f^{-1}(x) = (x+6)^{\frac{1}{3}}$ i $f^{-1}(x) = \frac{x^{\frac{1}{3}}}{2}$ 3 a $x \neq 3, x \in \mathbb{R}$ b $x \neq -4, x \in \mathbb{R}$ c $x \neq \frac{1}{2}, x \in \mathbb{R}$
- 3 d $x \geq 0, x \in \mathbb{R}$ e $x \geq 0, x \in \mathbb{R}$ f $x \geq \frac{1}{2}, x \in \mathbb{R}$ g $0 \leq x \leq \pi$ 4 a i $x \neq 6$ ii $f^{-1}(x) = \frac{6x+1}{x}$ b i $x \neq -7$ ii $f^{-1}(x) = \frac{3-7x}{x}$
- 4 c i $x \neq \frac{2}{3}$ ii $f^{-1}(x) = \frac{2x+5}{3x}$ d i $x \neq 2$ ii $f^{-1}(x) = \frac{2x-7}{x}$ e i $x \neq \frac{9}{4}$ ii $f^{-1}(x) = \frac{9x+8}{4x}$ f i $x \neq -\frac{6}{5}$ ii $f^{-1}(x) = \frac{4-6x}{5x}$
- 4 g i $x > 0$ ii $f^{-1}(x) = \sqrt{\frac{x}{6}}$ h i $x > 0$ ii $f^{-1}(x) = \sqrt{x+4}$ i i $x \geq 0$ ii $f^{-1}(x) = \sqrt{\frac{x-3}{2}}$ j i $x \geq 0$ ii $f^{-1}(x) = \sqrt{\frac{16-x}{3}}$
- 4 k i $x \geq 0$ ii $f^{-1}(x) = x^{\frac{1}{3}}$ l i \mathbb{R} ii $f^{-1}(x) = (\frac{x+5}{2})^{\frac{1}{3}}$ 5 a $h(x) = 3x - 6$ b $h^{-1}(x) = \frac{(x+6)}{3}$

Chapter 3 Exercise 4

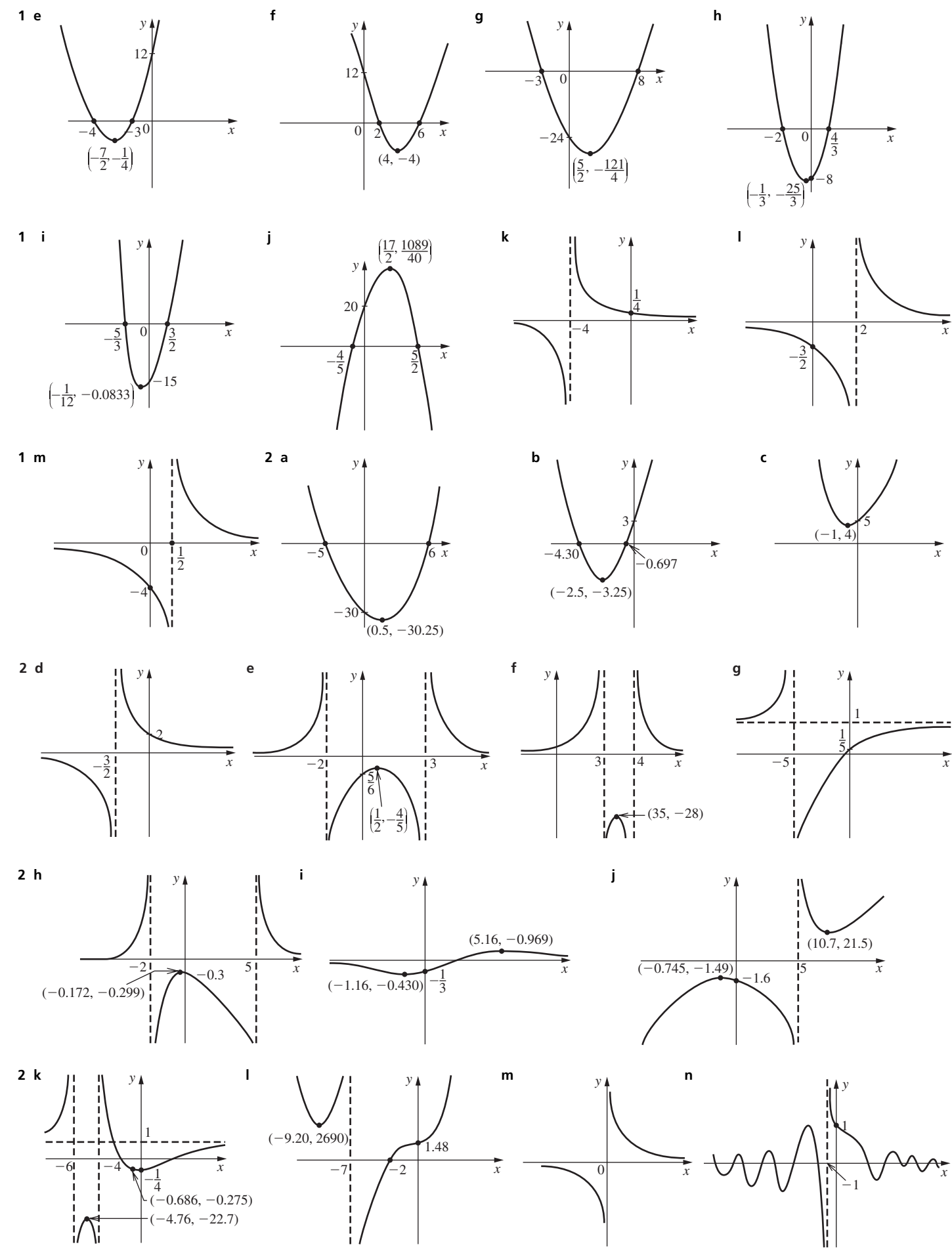
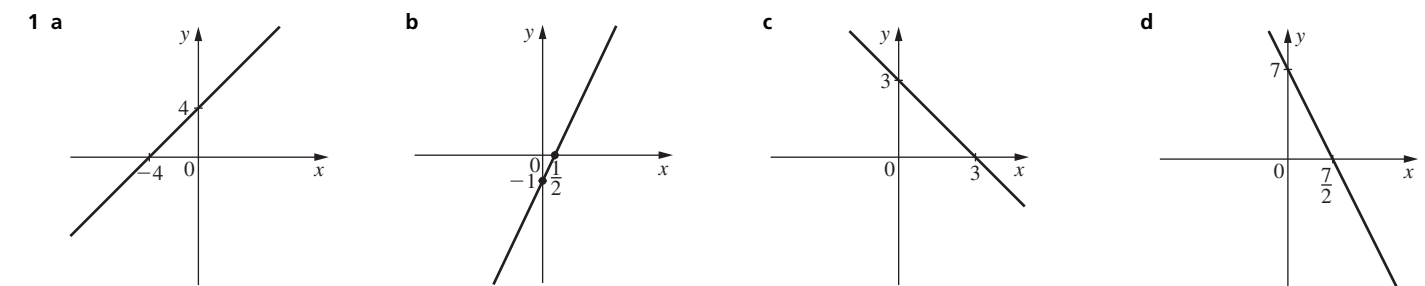




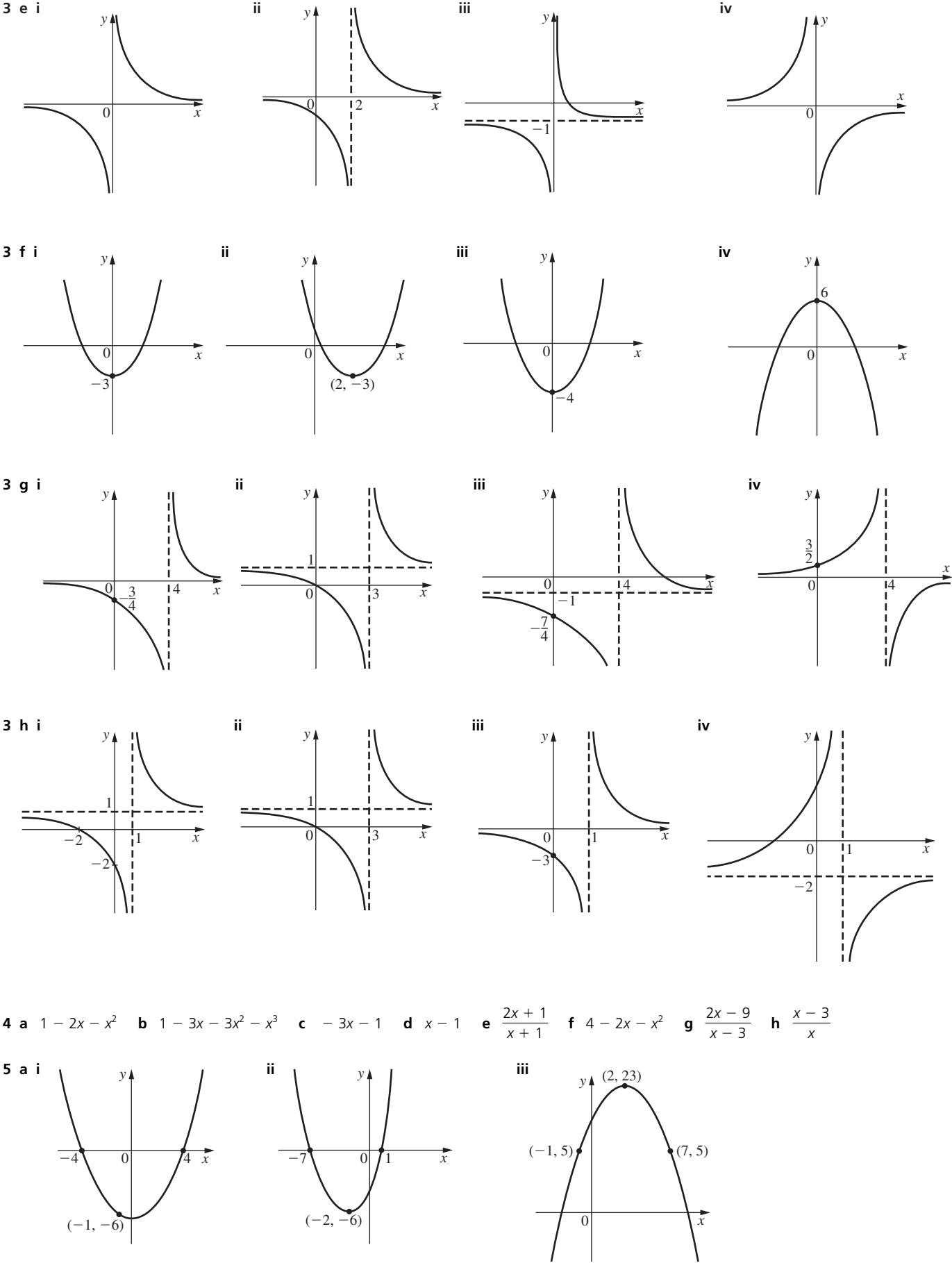
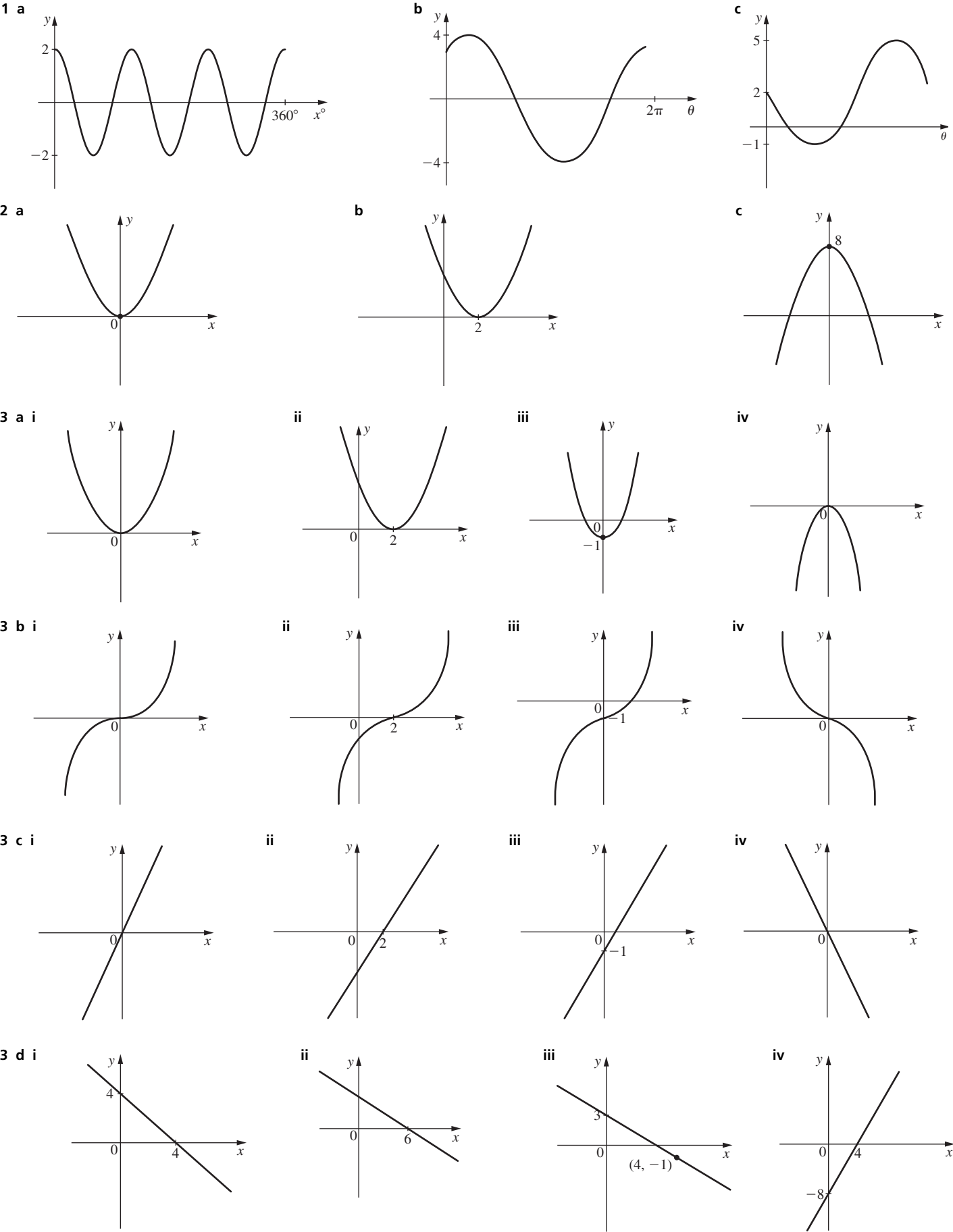
Chapter 3 Exercise 5

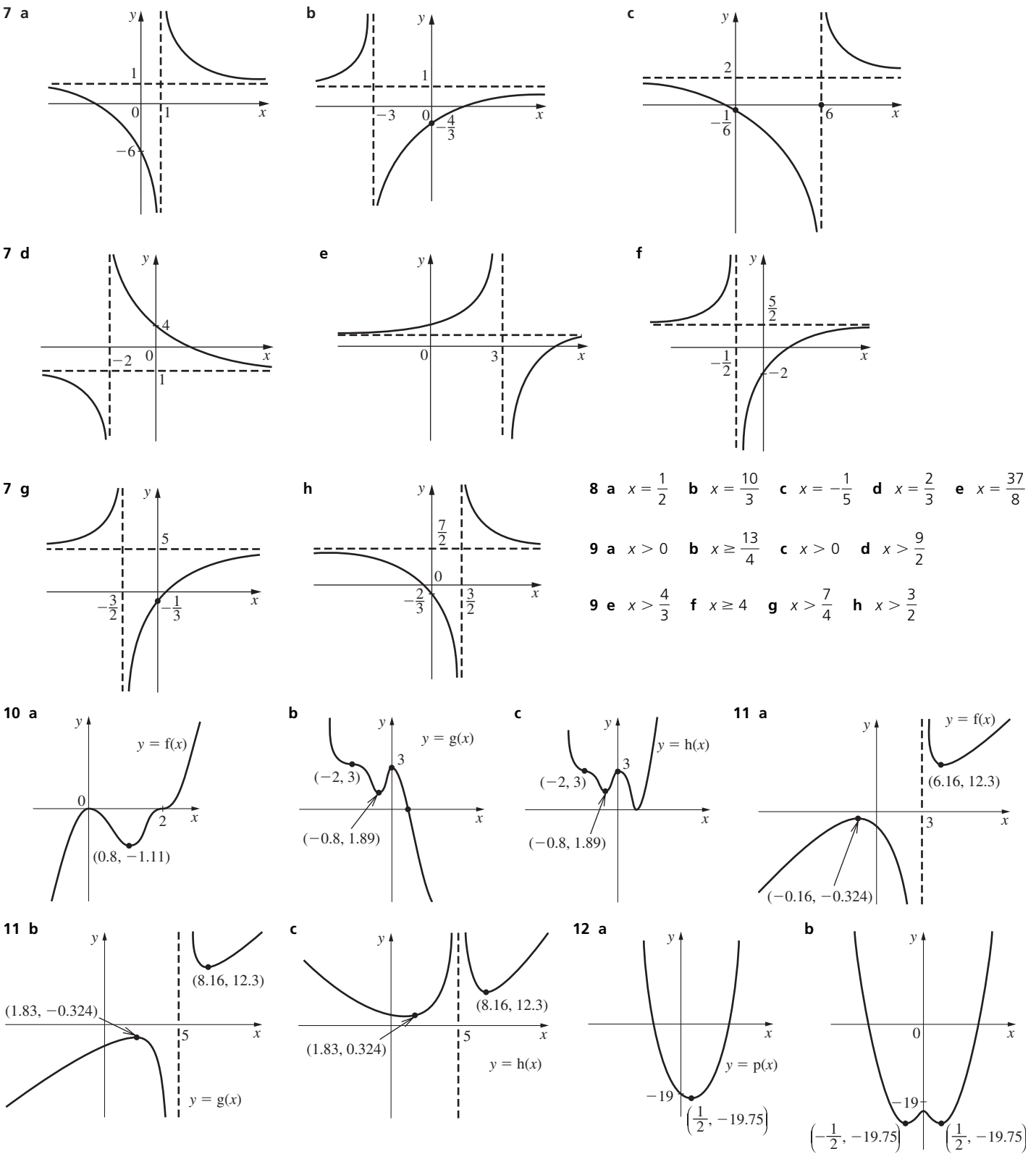
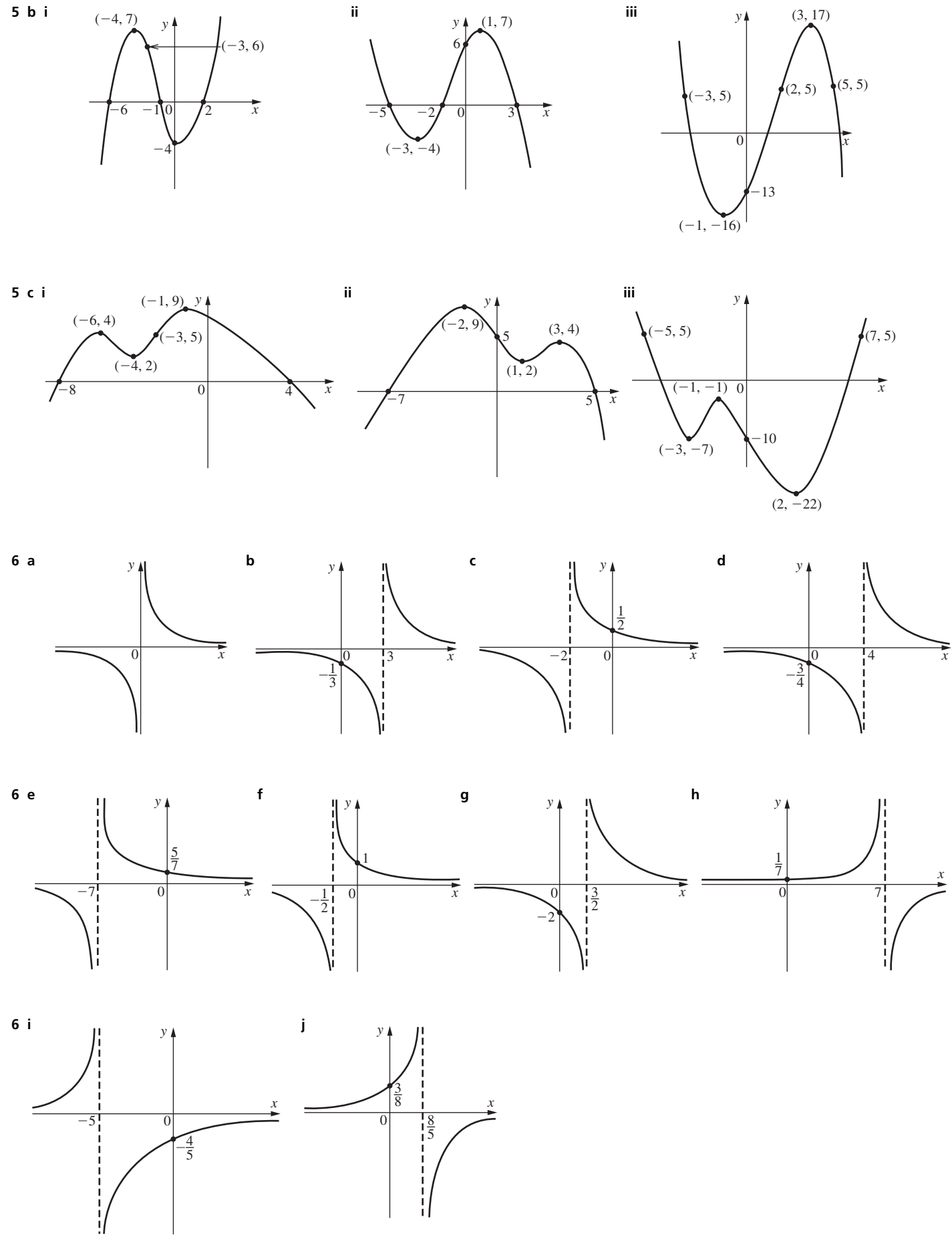


Chapter 3 Exercise 6

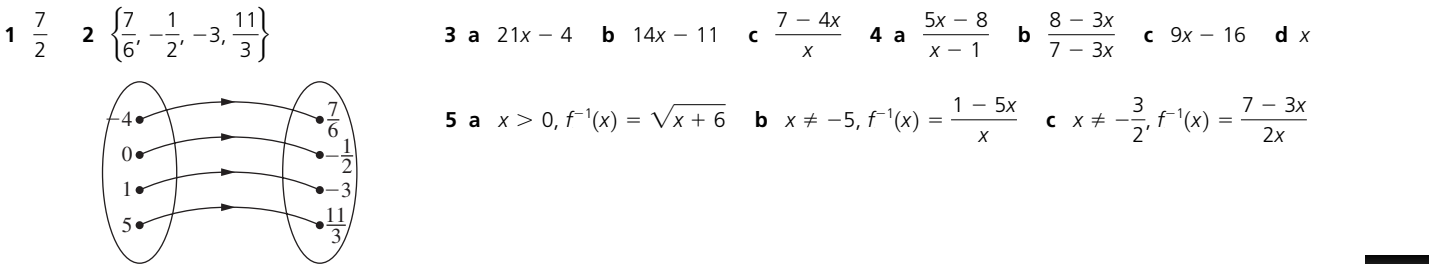


Chapter 3 Exercise 7

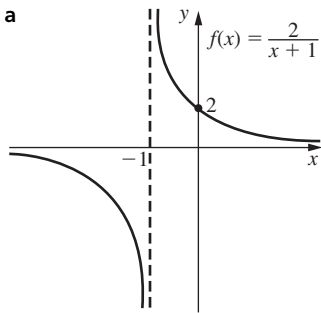




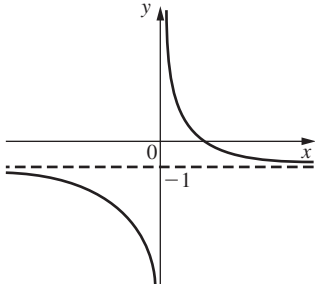
Chapter 3 Review Exercise



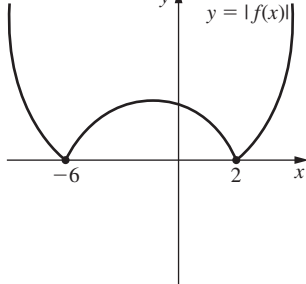
6 a



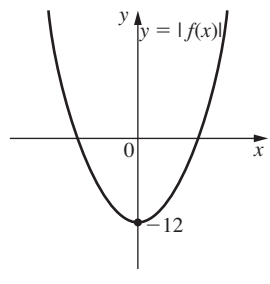
b



7 a

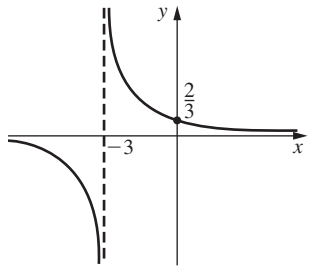


b

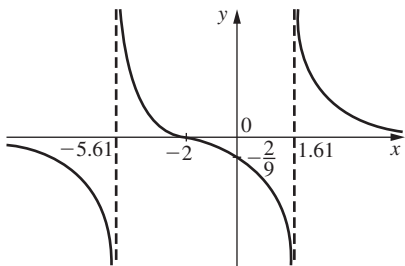


8 $x = -8, x = -1$ 9 $\frac{4}{5} < x < 2$

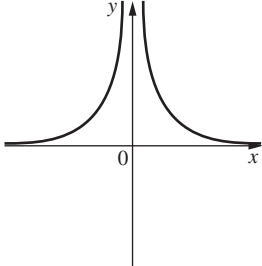
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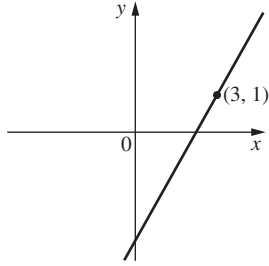
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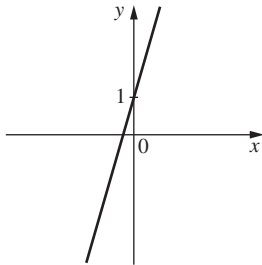
12



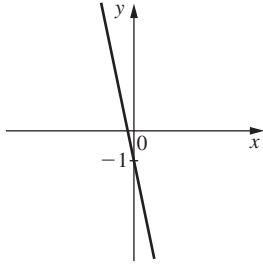
13 a



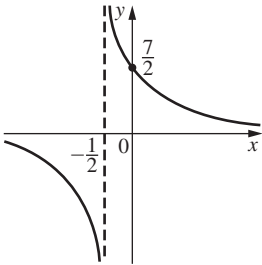
13 b



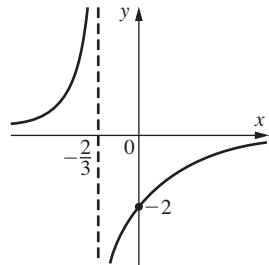
c



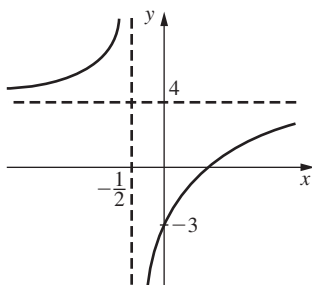
14 a



b

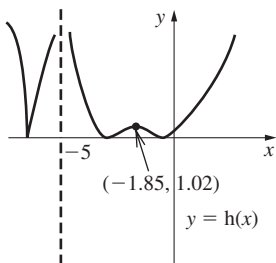
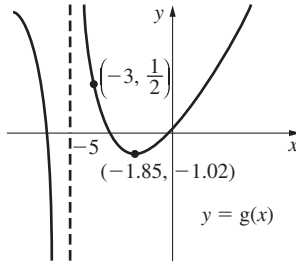
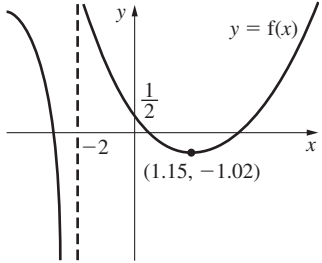


14 c



15 $x = \frac{19}{15}$ 16 $x > 0$

17



18 a $0 < y < 2$ b $f^{-1}(x) = \frac{-2x-1}{x-2}$ 19 $-3 \leq x \leq \frac{1}{3}$ 20 $f^{-1}(x) = \sqrt{\frac{-x-1}{x-1}}$

21 a $-\frac{1}{4} \leq x \leq \frac{1}{4}$ b $0 \leq y < \infty$ 22 $x < -1$ and $x > 4$

Chapter 4 Exercise 1

1 14 2 -47 3 21 4 a 51 b 1224 c 674 d -70 5 $\frac{9}{4}$ 6 a $Q(x) = x + 8, R = 13$ b $Q(x) = x^2 - 3x + 2, R = 1$

6 c $Q(x) = 2x^2 + 13x + 70, R = 427$ d $Q(x) = x^2 + x - 5, R = 11$ e $Q(x) = x^3 - x^2 - 4x + 7, R = 0$

6 f $Q(x) = \frac{x^4}{2} + \frac{x^3}{4} + \frac{x^2}{8} - \frac{7x}{16} - \frac{87}{32}, R = -\frac{439}{32}$ g $Q(x) = \frac{x^2}{2} - \frac{x}{4} - \frac{27}{8}, R = \frac{99}{8}$ h $Q(x) = -\frac{3x^3}{4} - \frac{7x^2}{16} - \frac{59x}{64} + \frac{177}{256}, R = \frac{2797}{256}$

7 a $f(x) = (3x - 1)(x - 2)$ b $f(x) = (x^2 + 11x + 47)(x - 5) + 242$ c $f(x) = (4x^2 - 5x + 6)(x + 3) - 35$

7 d $f(x) = (5x^4 - 20x^3 + 76x^2 - 304x + 1219)(x + 4) - 4878$ e $f(x) = (2x^5 - 2x^4 - 3x^3 + 3x^2 - 3x + 3)(x + 1) + 6$

7 f $f(x) = \left(\frac{x^2}{2} - \frac{13x}{4} + \frac{3}{8}\right)(2x - 1) - \frac{13}{8}$ g $f(x) = \left(x^3 - \frac{x^2}{2} - \frac{7x}{4} + \frac{7}{8}\right)(2x - 1) + \frac{81}{8}$

Chapter 4 Exercise 2

7 (c) (d) and (f) 8 a $(x - 1)^2(x + 1)$ b $(x - 1)(x - 2)(x + 3)$ c $(x + 2)(x - 1)(x - 5)$ d $(x + 1)(x - 1)(x^2 + 1)$

8 e $(2x - 1)(x + 3)(x - 4)$ f $(2x + 1)(x + 4)(x + 6)$ g $(2x + 3)(2x + 1)(3x - 4)$ h $(x - 3)(x + 3)(x^2 + 2)$ i $(x + 3)(x^2 + 1)(2x^2 + 5)$

8 j $(6x - 1)(3x + 4)(2x + 5)(x^2 + 4)$

Chapter 4 Exercise 3

1 17 2 $\frac{133}{16}$ 3 $p = 24$ 4 $k = 2$ 5 $k = 10, (x - 3)(x - 1)(2x - 1)$ 6 $a = 2, (x + 2)(x - 3)(x + 3)$ 7 $p = 0, q = -1$

8 $k = -46, (2x + 1)(x - 4)(x + 6)$ 9 $p = 11, q = -21$ 10 $k = 5$

Chapter 4 Exercise 4

1 $x = -1, x = -3$ 2 $x = -\frac{1}{2}, x = 6$ 3 $x = -2, x = 5, x = \frac{1}{2}, x = 4$ 4 a $x = -1, x = 3, x = 4$ b $x = -7, x = -2, x = 2$

4 c $x = -11, x = -3$ d $x = -3, x = -2, x = 4, x = 5$ e $x = -2, x = 4$ f $x = -2, x = \frac{1}{2}, x = 8$ g $x = -\frac{2}{3}, x = \frac{1}{2}, x = \frac{3}{2}$

5 $x = -2, x = 1, x = 9$ 7 $x = 3$ 8 a $p = 30$ b $x = 3, x = 5$ 9 a $k = 83$ b $x = \frac{1}{2}, x = 7$

10 a $1 \leq x \leq 7$ b $2 \leq x \leq 5$ c $0 \leq x \leq 1$ d $3 \leq x \leq 7$ e $\frac{1}{2} \leq x \leq \frac{10}{3}$ 11 $6 < t < 10$

Chapter 4 Exercise 5

1 $y = x^2 + 3x - 4$ 2 $y = -x^2 + 6x$ 3 $y = 10 + 3x - x^2$ 4 $y = 2x^2 - 4x - 6$ 5 $y = 3x^2 - 12x + 12$ 6 $y = x^3 + 2x^2 - 11x - 12$

7 $y = \frac{1}{2}x^3 - \frac{3}{2}x^2 - 5x + 12$ 8 $y = -2x^3 + 2x^2 + 28x - 48$ 9 $y = x^4 - 2x^3 - 13x^2 + 14x + 24$

10 $y = 20x^4 - 180x^3 + 420x^2 + 20x - 600$ 11 $y = -2x^4 + 6x^3 + 18x^2 - 46x + 24$

Chapter 4 Exercise 6

3 $x + 3$ 4 $2x + 1$ 5 $3x - 11, 15$ 7 $x + 5 + \frac{-14x - 21}{x^2 + 5}$ 8 $x - \frac{9}{2} + \frac{-30x + 85}{2(x^2 + 7)}$

9 $3x^3 - 6x^2 + 10x - 20 + \frac{47}{x + 2}$ 10 $2x^3 + 6x + \frac{18x + 13}{x^2 - 3}$ 11 $-\frac{x^3}{2} + \frac{x^2}{4} + \frac{27x}{8} - \frac{67}{16} + \frac{211}{16(2x + 1)}$ 12 $-x^4 + x^2 + 5x - 1 + \frac{12 - 5x}{x^2 + 1}$

13 $x = -4, x = -3, x = 1, x = 2$ 14 $x = -2, x = 2, x = -6$

Chapter 4 Exercise 7

1 $x = -1, x = \frac{1}{2}, x = 3$ 2 $x = -7, x = -1, x = -\frac{1}{2}, x = 4$ 3 $x = -2, x = -\frac{1}{2}, x = 3$ 4 $x = 3$ 5 $x = -4.29, x = -0.428, x = 2.72$

6 $x = 5.50$ 7 $x = 0.388, x = 1$ 8 $x = 4.92, x = 1.02, x = -2.19, x = -3.74$ 9 $x = 10.2, x = 0.203, x = -1.44$

10 $x = -0.953$ 11 $f(x) = -4x^3 + 10x^2 + 28x - 16, x = -2, x = 0.388, x = 3.86$ 12 $(x + 5)(x - 1)(x - 4)$

13 $(3x + 4)(3x - 4)(2x - 1)(2x + 1)$ 14 a $x < -0.303$ and $1 < x < 3.30$ b $-3.42 < a < 1.27$

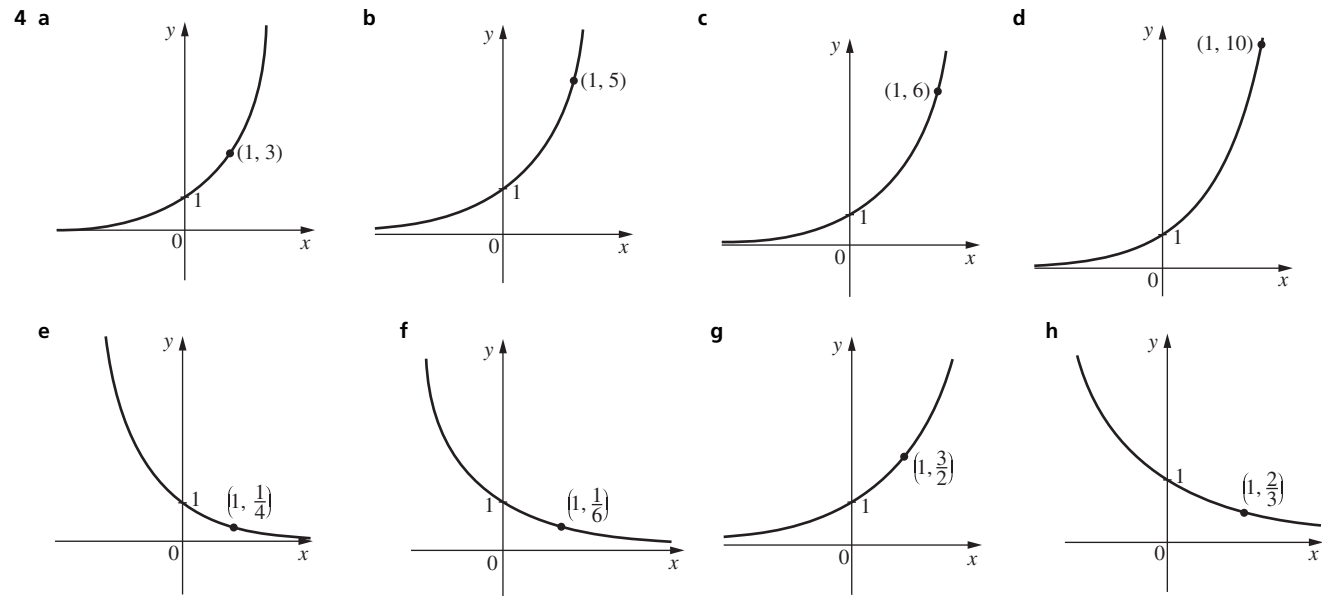
15 a $-0.961 < x < 1.63$ and $x < 3.83$ b $7.89 < a < 29.1$

Chapter 4 Review Exercise

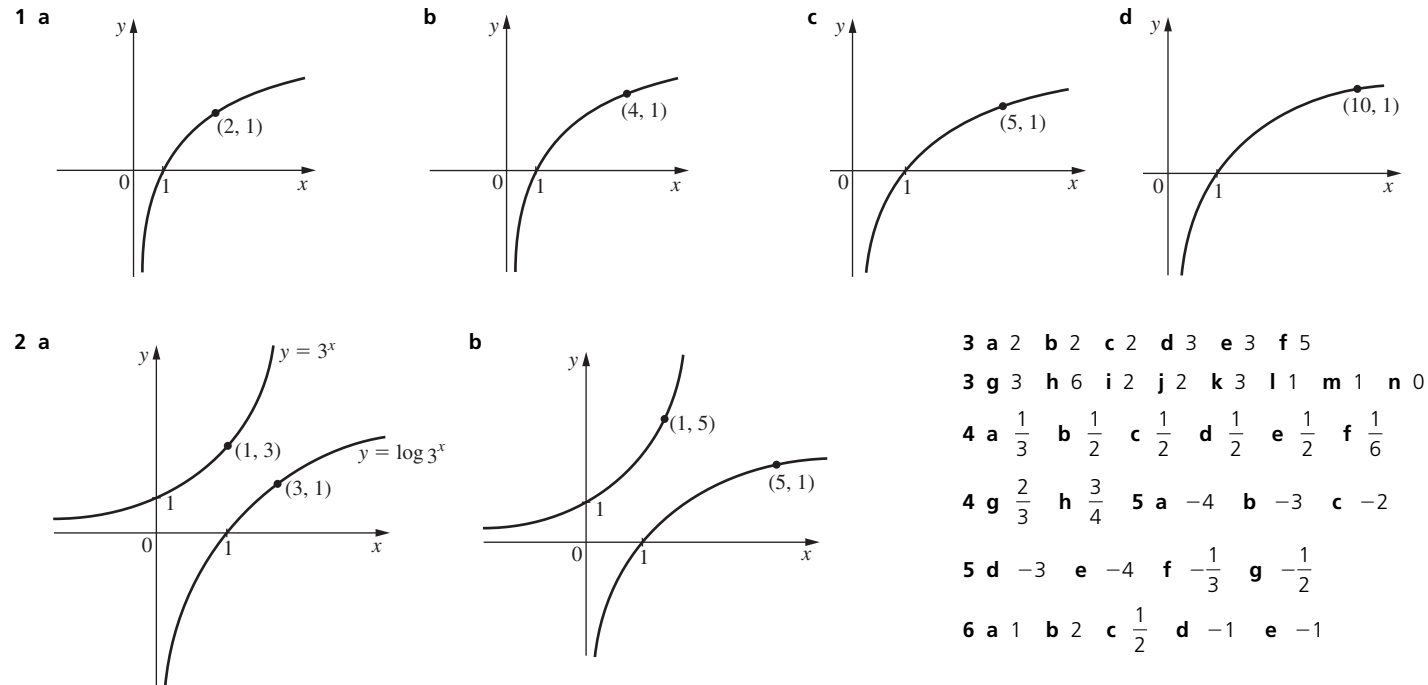
- 1 -35 2 $3x^3 + 4x^2 + 8x + 22 + \frac{45}{x-2}$ 3 $f(x) = \left(x^4 + \frac{x^3}{2} + \frac{x^2}{4} + \frac{17x}{8} + \frac{17}{16}\right)(2x-1) - \frac{95}{16}$ 4 $(x-3), (x-9)$
5 $f(x) = (2x+3)(x-4)(x+1)$ 6 $g(x) = (2x-1)(x-5)(x+2)(3x-2)$ 7 $k(x) = (x-4)(x+1)(x^2+5)$ 8 $x = -7, x = 3$
9 $x = -4, x = \frac{1}{2}, x = 1$ 10 $f(x) = 2(x+3)(2x-1)(x-4)^2$ 11 $x = -1.47$ 12 $x = 6.59, x = 2.38, x = -1.97$
13 $\frac{x^2}{2} - \frac{11x}{4} + \frac{35}{8} - \frac{67}{8(2x-1)}$ 14 $x^2 - 5 + \frac{3x}{x^2+1}$ 15 $a = -\frac{4}{5}, b = -\frac{6}{5}$ 16 $a = -6$ 17 $a = 4$ 18 $k = 6$

Chapter 5 Exercise 1

- 1 a p^9 b p^5 c x^{15} d $21y^5$ e $16x^{12}$ f t^2 g $2p^2$ h $6p^7$ 2 a 4 b 3 c $\frac{1}{10}$ d 1 e 125 f $\frac{1}{3}$ g $\frac{1}{4}$ h $\frac{1}{8}$ i 9
3 a x^6 b $\frac{4}{3}y^4$ c $10p^{-6}$ d t^2 e $6m^{-\frac{2}{3}}$ f $12x^5 + 15x$ g $2x + 1$ h $x + 2 + x^{-1}$



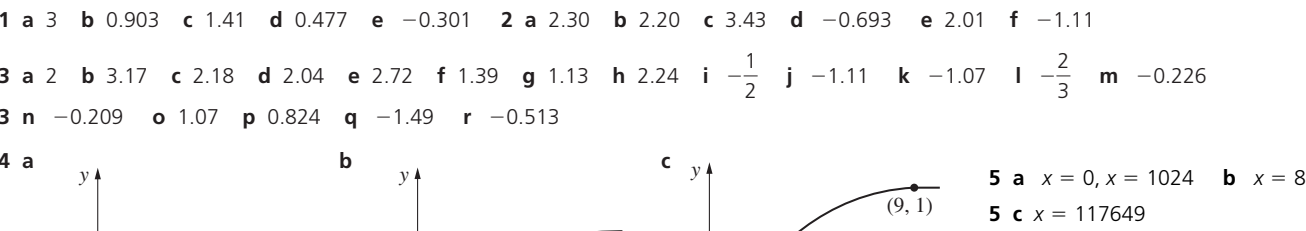
Chapter 5 Exercise 2



Chapter 5 Exercise 3

- 1 a $\log_a 18$ b $\log_a 15$ c $\log_a 5$ d $\log_a 64$ e $\log_a 24$ f $\log_a 4$ g $\log_a 32$ h $\log_a 6$ i $\log_a \frac{1}{8}$ j $\log_a 72$ k $\log_a 12$
2 a $\log_3 15$ b $\log_2 \left(\frac{5}{2}\right)$ c $\log_2 \left(\frac{8}{9}\right)$ d $\log_a \left(\frac{xy^2}{t^3}\right)$ 3 a 2.699 b 2 c 1 d $\frac{1}{2}$ e 2 f 2 g 1 h 0 i -3 j 4 k $-\frac{5}{2}$
4 a $\log_a 3x^3$ b $\log_a \left(\frac{2}{x}\right)$ c $\log_a (x-1)$ d $\log_a \left(\frac{(x+2)^2}{3}\right)$ 5 a 2 b $\frac{1}{2}$ c $\frac{3}{2}$ d $\frac{2}{3}$ e 2 f $\frac{1}{2}$ $\log_x y = \frac{1}{\log_y x}$ 6 $y = 4x^3$
7 $y = 9x^4$ 8 $y = px^5$ 10 a $x = 7$ b $x = 57$ c $x = 6$ d $x = 9$ e $x = 8$ f $x = \frac{1}{6}$ g $x = 25$ h $x = 9$
11 a $x = 2$ b $x = 4$ c $x = 3$ d $x = 15$ e $x = \frac{26}{3}$ f $x = 10$ 12 $S_1 = 400$

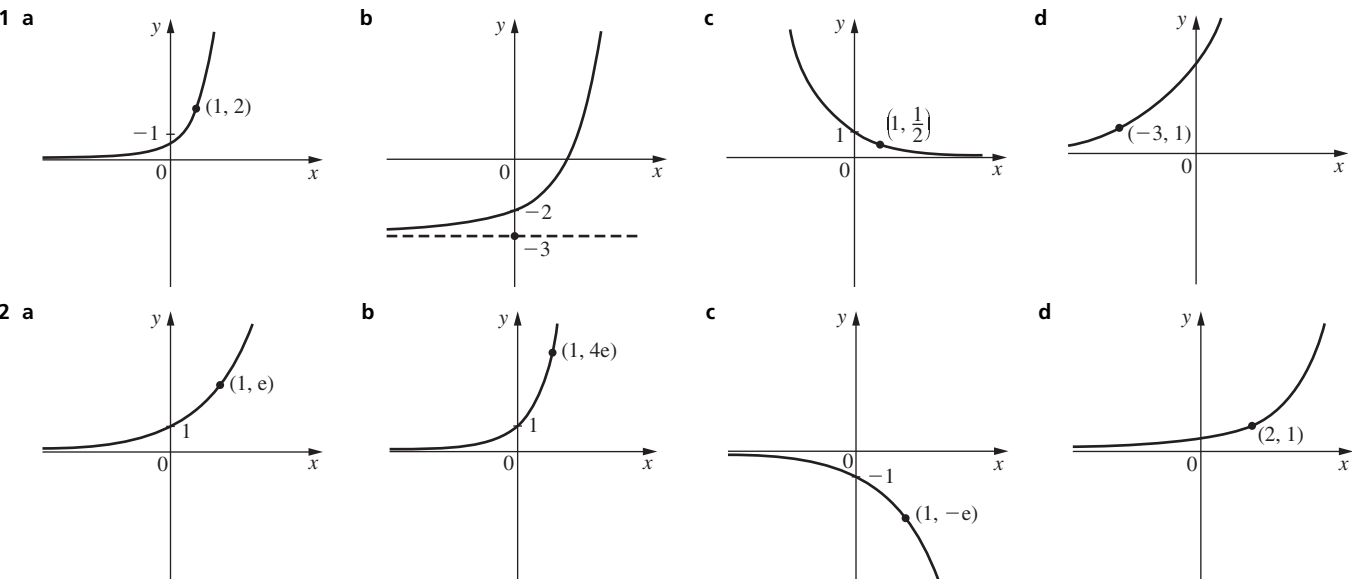
Chapter 5 Exercise 4

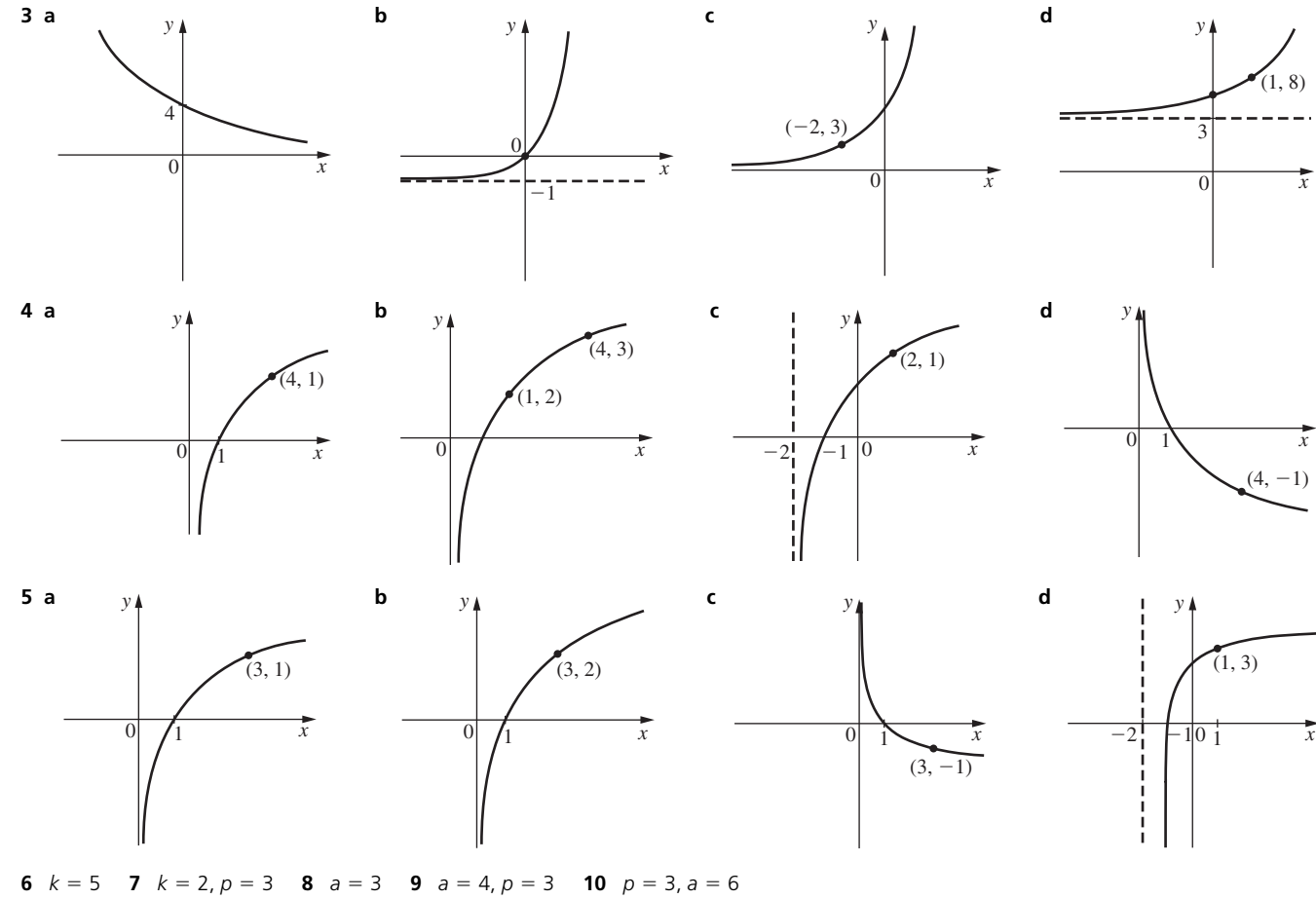


Chapter 5 Exercise 5

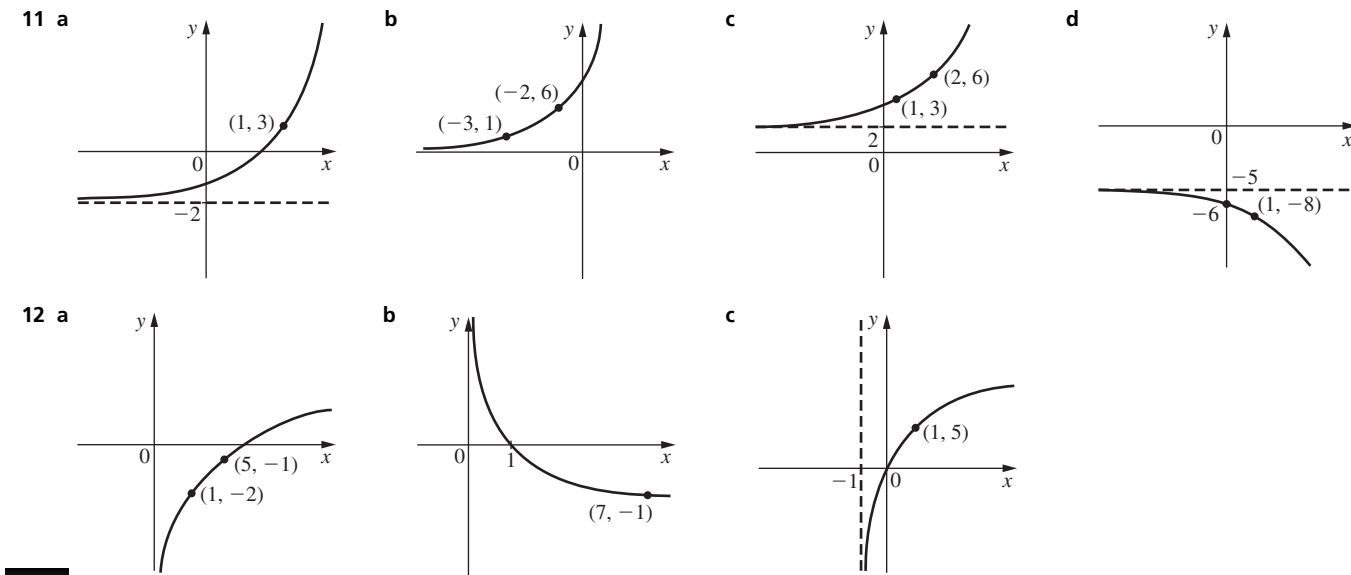
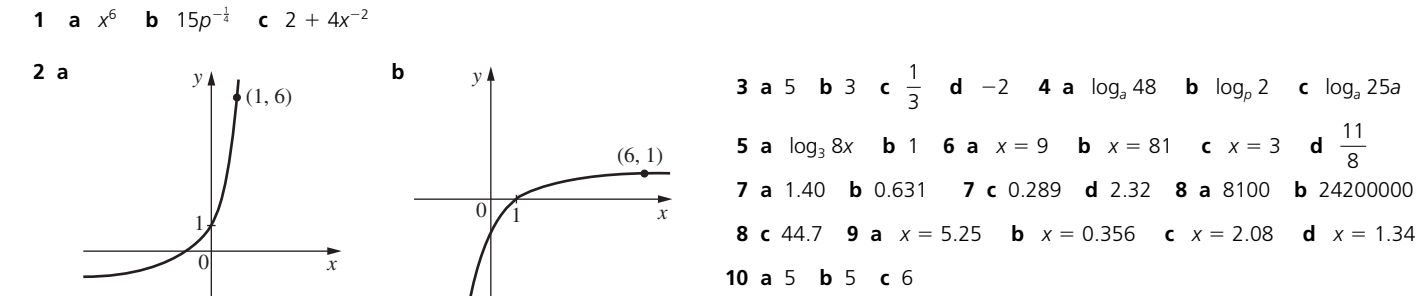
- 1 a $x = 8$ b $x = 3.36$ c $x = 1.86$ d $x = 3.53$ e $x = 0.862$ f $x = 0.416$ 2 a $x = 2.48$ b $x = 3.40$ c $x = 5.60$
2 d $x = 1.50$ e $x = -0.981$ 3 a $x = 8100$ b $x = 7.39$ c $x = 22000$ d $x = 8890000$ e $x = 1.22$
4 a $x = 9$ b $x = 6$ c $x = 4$ d $x = 6$ 5 a 40 b 132 c 3.84 days 6 37.8 months 7 a 80°C b 10.8 mins
8 a 2100g b 1650g c 57.8 years 9 a 20100 km b 22.7 years 10 a 220 b 57 c 2027
11 a $k = 0.0133$ b 67.2 hours c 9.6 hours longer 12 a $k = 0.0114$ b 60.8 years 13 $\frac{\ln 12}{\ln 5}$ 14 $\frac{\ln 2}{\ln 6}$ 15 $\frac{\ln 2}{\ln 80}$ 16 $\frac{\ln \frac{64}{3}}{\ln \frac{9}{2}}$
17 $x = 2 - \log_2 5$ 18 $x = -1 + \log_4 3$ 19 $x = -\frac{3}{2} + \log_4 5$ 20 $x = \log_6 4$ 21 $x = 0$ 22 $x = 0.631$

Chapter 5 Exercise 6





Chapter 5 **Review Exercise**



13 $k = 7$ **14** $p = 3, q = 5$ **15 a** $k = 0.0633$ **b** Yes as it will be ok for 8.07 hours **16** $x = \pm 6$ **17** $\frac{\ln 9}{\ln 8}$ **18** $-1 + \log_5 3$

19 $x = 4, y = 4$ **20** $x = 1, y = 0$ **21** $x = \frac{15}{7}, y = \frac{10}{7}$ **22 a** $x < -\frac{2}{3}, x > \frac{3}{2}, x \in \mathbb{R}$ **b** $y \geq 0, y \in \mathbb{R}$ **23** $x = \pm 8$

24 a $k = \frac{3}{2}, m = 1$ **b** $x = \sqrt{8}$

Chapter 6 **Exercise 1**

1 $u_n = 2n + 3$ **2** $u_n = 5n - 4$ **3** $u_n = 6n + 2$ **4** $u_n = -9n + 69$ **5** $u_n = -4n + 8$ **6** $u_n = 11n - 4, u_{20} = 216$

7 $u_n = 110n + 90, u_{13} = 1520$ **8** $u_n = -7n + 24, u_{19} = -109$ **9** $u_n = \frac{1}{2}n + \frac{1}{2}, u_{15} = 8$ **10** $n = 143$ **11** $n = 23$ **12** $n = 27$

13 $u_n = 4n - 3$ **14** $u_n = 9n + 7$ **15** $u_n = -6n + 50$ **16** $u_n = -\frac{1}{2}n - 4$ **17** $k = 2$ **18** $k = 8$ **19** $k = 3$ **20** $k = -9, k = 6$

Chapter 6 **Exercise 2**

1 $\frac{n}{2} + \frac{3}{2}n^2$ **2** $n^2 + 7n$ **3** $-\frac{3}{2}n^2 + \frac{163n}{2}$ **4** $2014n - 6n^2$ **5** $\frac{n}{3} + \frac{n^2}{6}$ **6** 203 **7** 354 **8** 1050 **9** 5586 **10** $\frac{n(n+1)}{2}$

11 n^2 **12** 4, 10, 16 **13** $n = 16$ **14** $n = 6$ **15** 1344 **16 a** $d = 2$ **b** 308

Chapter 6 **Exercise 3**

1 $u_6 = \frac{1}{4}, u_n = 2^{4-n}$ **2** $u_6 = \frac{5}{64}, u_n = 80\left(\frac{1}{4}\right)^{n-1}$ **3** $u_6 = 486, u_n = 2(3)^{n-1}$ **4** $u_6 = -160, u_n = 5(-2)^{n-1}$ **5** $u_6 = \frac{-25}{8}, u_n = 100\left(-\frac{1}{2}\right)^{n-1}$

6 $u_6 = 384, u_n = 12(2)^{n-1}$ **7** $u_6 = 18750, u_n = 6(5)^{n-1}$ **8 a** $\frac{255}{16}$ **b** 107 **c** 6560 **d** -425 **e** 66.4 **f** 3060 **g** 585936

9 $S_n = \frac{x(1-x^n)}{1-x}$ **10** $S_n = \frac{1-(-x)^n}{1+x}$ **11** $S_n = \frac{(1-(-3x)^n)}{1+3x}$ **12** $u_n = 5(2)^{n-1}$ **13** $u_n = 270\left(\frac{1}{3}\right)^{n-1}$ **14** $u_n = 4(-3)^{n-1}$

15 $u_n = \frac{1}{8}(-4)^{n-1}$ **16** $k = -2$ or $k = 10$ **17** $k = 1$ or $k = 3$ **18** $k = 8$ **19** $n = 9$ **20** $n = 5$ **21** $n = 8$ **22** $a = \frac{4}{3}, r = -4$

Chapter 6 **Exercise 4**

1 $S_\infty = 40$ **2** $S_\infty = \frac{243}{2}$ **3** Does not converge **4** $S_\infty = -\frac{512}{13}$ **5** Does not converge **6** $S_\infty = 12$ **7** $S_\infty = 300$ **8** $S_\infty = 50$

9 $S_\infty = \frac{36}{7}$ **10** $-1 < x < 1$ **11** $x < -1$ or $x > 1$ **12** $\frac{58}{9}$ **13** $\frac{214}{99}$ **14** $\frac{727}{99}$ **15 a** 1950 **b** $\frac{58025}{32}$ **c** $\frac{32}{9}$

Chapter 6 **Exercise 5**

1 28 days **2** \$2720.98 **3** 98691 dkk **4** 20 years **5** 4.3% **6** £ 8820.36 **7 a** 17623 rats **b** 14.2 months **8 a** 187 leopards **b** 2012

9 a 3.07m **b** 10

Chapter 6 **Exercise 6**

1 a 35 **b** 252 **c** 896 **2 a** $\sum_{r=1}^5 4r$ **b** $\sum_{r=1}^{n+2} 5r - 7$ **c** $\sum_{r=1}^\infty 4r + 5$

3 a $3n^2 + n$ **b** $\frac{2}{3}n^3 + \frac{3}{2}n^2 + \frac{17}{6}n$ **c** $-\frac{8}{3}n^3 - 2n^2 + \frac{53}{3}n$ **d** $\frac{1}{2}(k+1)(7k+8)$

Chapter 6 **Exercise 7**

1 a 30 **b** 336 **c** 56 **d** 126 **e** 70 **2** 360 **3** 2002 **4 a** 39916800 **b** 990 **c** 330 **5** 1712304 **6** 19068840 **7** 12870

8 a 216 **b** 120 **9 a** $n = 6$ **b** $n = 5$ **c** $n = 4$ **d** $n = 6$ **10 a** $n = 4$ **b** $n = 10$ **c** $n = 9$ **11 a** $n = 7$ **b** $n = 3$ **c** $n = 3$

Chapter 6 Exercise 8

- 1 a** $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$ **b** $729x^6 + 2916x^5 + 4860x^4 + 4320x^3 + 2160x^2 + 576x + 64$ **c** $1 - 4x + 6x^2 - 4x^3 + x^4$
- 1 d** $32p^5 - 240p^4q + 720p^3q^2 - 1080p^2q^3 + 810pq^4 - 243q^5$ **2 a** $x^3 + 3x + 3x^{-1} + x^{-3}$ **b** $x^5 + 10x^3 + 40x + \frac{80}{x} + \frac{80}{x^3} + \frac{32}{x^5}$
- 2 c** $x^6 - 6x^4 + 15x^2 - 20 + \frac{15}{x^2} - \frac{6}{x^4} + \frac{1}{x^6}$ **d** $16t^4 - 8t^2 + \frac{3}{2} - \frac{1}{8t^2} + \frac{1}{256t^4}$ **3** $x^6 + 9x^5 + 30x^4 + 45x^3 + 30x^2 + 9x + 1$
- 4 a** 40 **b** 7000 **c** 3840 **d** 6480 **e** 150994944 **f** 21 **g** -8 **h** 90720 **5 a** 58 **b** -1392 **c** -243 **d** 5
- 6 a** $x^7 - x^6 - 69x^5 + 109x^4 + 1616x^3 - 3360x^2 - 12800x + 32000$ **b** $x^6 - 6x^5 + 15x^4 - 26x^3 + 39x^2 - 42x + 37 - \frac{30}{x} + \frac{12}{x^2} - \frac{8}{x^3}$
- 6 c** $x^7 - 2x^5 - 6x^3 + 8x + \frac{17}{x} - \frac{6}{x^3} - \frac{20}{x^5} - \frac{8}{x^7}$ **7 a** 22 **b** 7688 **c** -5888 **8** 1360 **9 a** 1.04 **b** 0.210 **c** 15800000
- 10** $-22688x^2 + 12480x - 3200$ **11** Proof: $p^2 = -\frac{1}{5}$

Chapter 6 Review Exercise

- 1** -7 **2 a** $r = 4$ **b** $S_n = 16(4^n - 1)$ **3 a** $S_n = \frac{3n^2}{2} + \frac{n}{2}$ **b** $n = 30$ **4 a** $r = \frac{2}{3}$ **b** $a = 9$
- 5 a** 1, 9 **b** $u_n = 4n - 3$ **6 a** $|x| < \frac{3}{2}$ **b** 5 **7 a** $8n - 3$ **b** 50 **8 a** $a = \pm 3$ **9 a** $a = 9$ **10 a** $n = 6$ **b** $u_n = 15(4)^{n-1}$
- 11 a** $a = 2, b = -3$ **12 a** $x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$ **b** 32.8080401 **13 a** $r = \frac{1}{2}$ **b** $d = -\frac{9}{20}$ **14 a** 59 **b** $n = 12, d = 0.25$
- 14 c i** 99 **ii** 100 **15** 280 and 84 **16 k** $= \frac{36}{p^3}$ **17** 34642080 **18** 4455 **19** $6n - \frac{5}{6}n(n + 1)(2n + 1), -972$ **20 n** = 10

Chapter 7 Exercise 1

- 1 a** $\pm \frac{\sqrt{3}}{2}$ **b** $\pm \frac{1}{\sqrt{2}}$ **c** $\pm \frac{2\sqrt{6}}{7}$ **d** No possible value **2 a** $\pm \frac{1}{2}$ **b** $\pm \frac{3}{5}$ **c** No possible value **d** $\pm \frac{2\sqrt{6}}{5}$ **4** $\pm \frac{1}{3}$
- 5 a** $\sin \theta$ **b** $\tan \theta$ **c** $-\sin \theta$ **d** $7 \cot^2 \theta$ **7 a** 0.464, 2.68, 3.61, 5.82 **b** $\frac{\pi}{2}, \frac{3\pi}{2}$ **c** 0.785, 2.90, 3.93, 6.04
- 7 d** $\frac{\pi}{4}, \frac{5\pi}{4}$ **e** 3.53, 5.90 **f** 1.05, π , 5.24

Chapter 7 Exercise 2

- 1 a** $\frac{1 + \sqrt{3}}{2\sqrt{2}}$ **b** $\frac{\sqrt{3} - 1}{2\sqrt{2}}$ **c** $\frac{1 + \sqrt{3}}{\sqrt{3} - 1}$ **2 a** and **b** $\frac{\sqrt{3} - 1}{2\sqrt{2}}$ **3** $\frac{1 - \sqrt{3}}{2\sqrt{2}}$ **4** $-\frac{(1 + \sqrt{3})}{2\sqrt{2}}$ **8 a** 0 **b** $\frac{\sqrt{3}}{2}$
- 9 a** $\frac{4}{5}$ **b** $\frac{117}{125}$ **c** $-\frac{44}{117}$ **10** $-\frac{15}{65}$ **11** 0.951 **12** $-\frac{7}{2}$ **13** 0.2829, -0.3877 **14** 52.5°, 232.5° **16** $\frac{6}{10}$ **17 a** $\frac{24}{25}$ **b** $-\frac{7}{25}$
- 17 c** $\frac{336}{625}$ **d** $\frac{527}{625}$ **18 a** 40.9°, 220.9° **b** 0°, 180° **c** 7.6°, 187.6° **19 a** 2.88, 6.02 **b** 2.36, 5.50 **c** 1.70, 4.84

Chapter 7 Exercise 3

- 1 a** $\frac{\sqrt{3}}{2}$ **b** $\frac{1}{2}$ **c** $-\sqrt{3}$ **2** $\frac{120}{169}$ **3** $\frac{161}{289}$ **4 a** $\frac{1}{\sqrt{5}}$ **b** $\frac{2}{\sqrt{5}}$ **5** $3 \cos^5 \theta + 4 \cos^4 \theta - 4 \cos^3 \theta - 3 \cos^2 \theta$
- 6** $2 \cos \theta [16 \sin^5 \theta - 16 \sin^3 \theta + 3 \sin \theta]$ **7** $-\frac{119}{169}$

Chapter 7 Exercise 4

- 3** $\frac{\sqrt{3} + 1}{2\sqrt{2}}$ **4** $\frac{\sqrt{2 + \sqrt{2}}}{2}$ **5 a** $\frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}$ **b** $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **6 a** 30°, 90°, 150°, 270° **b** 0°, 180°
- 6 c** 60°, 90°, 270°, 300° **d** 90° **e** 60°, 300° **f** 60°, 300° **g** 120°, 240° **h** 210°, 330°
- 7 a** $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$ **b** $0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$ **c** $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ **d** $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$ **e** $0, \frac{2\pi}{3}, \frac{4\pi}{3}$ **f** π **g** $0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}$
- 8 a** $-\pi, 0$ **b** $-\frac{\pi}{3}, \frac{\pi}{3}$ **c** No solution **d** $-\pi, -\frac{2\pi}{3}, \frac{2\pi}{3}$ **9 a** 0.841, 1.82, 4.46, 5.44 **b** 0, 1.82, 4.46

Chapter 7 Exercise 5

- 1 a** $10 \cos(x - 53.1)^\circ$ **b** $13 \cos(x - 67.4)^\circ$ **c** $\sqrt{10} \cos(x - 288.4)^\circ$ **d** $\sqrt{5} \cos(x - 153.4)^\circ$ **2 a** $2 \cos\left(\theta - \frac{11\pi}{6}\right)$ **b** $\sqrt{2} \cos\left(\theta - \frac{7\pi}{4}\right)$
- 2 c** $\sqrt{5} \cos(\theta - 4.25)$ **d** $2 \cos\left(\theta - \frac{2\pi}{3}\right)$ **3 a** $17 \cos(x + 28.1)^\circ$ **b** $\frac{5}{2} \cos(x + 36.9)^\circ$ **4 a** $2 \sin\left(\theta + \frac{2\pi}{3}\right)$ **b** $\sqrt{2} \sin\left(\theta + \frac{3\pi}{4}\right)$
- 5 a** $\sqrt{10} \sin(x - 108.4)^\circ$ **b** $2 \sin(x - 240)^\circ$ **6 a** $\sqrt{2} \cos(2x - 45)^\circ$ **b** $2 \cos(3x - 300)^\circ$ **c** $6 \cos\left(\theta - \frac{11\pi}{6}\right)$ **d** $\sqrt{2} \cos\left(30\theta - \frac{3\pi}{4}\right)$
- 7 a** Minimum (135°, $-5\sqrt{2}$) Maximum (315°, $5\sqrt{2}$) **b** Minimum (150°, 3) Maximum (330°, 7)
- 7 c** Minimum (157.5°, -8.90), (337.5°, -8.90) Maximum (67.5°, 10.9), (247.5°, 10.9)
- 8 a** Minimum (5.33, $-\sqrt{12}$) Maximum (2.19, $\sqrt{12}$) **b** Minimum $\left(\frac{\pi}{18}, -2\right), \left(\frac{13\pi}{18}, -2\right), \left(\frac{25\pi}{18}, -2\right)$ Maximum $\left(\frac{7\pi}{18}, 2\right), \left(\frac{19\pi}{18}, 2\right), \left(\frac{31\pi}{18}, 2\right)$
- 8 c** Minimum $\left(\frac{5\pi}{24}, -5.83\right), \left(\frac{11\pi}{24}, -5.83\right), \left(\frac{17\pi}{24}, -5.83\right), \left(\frac{23\pi}{24}, -5.83\right)$ $\left(\frac{29\pi}{24}, -5.83\right), \left(\frac{35\pi}{24}, -5.83\right), \left(\frac{41\pi}{24}, -5.83\right), \left(\frac{47\pi}{24}, -5.83\right)$
- 8 c** Maximum $\left(\frac{\pi}{12}, -0.172\right), \left(\frac{\pi}{3}, -0.172\right), \left(\frac{7\pi}{12}, -0.172\right), \left(\frac{5\pi}{6}, -0.172\right)$ $\left(\frac{13\pi}{12}, -0.172\right), \left(\frac{4\pi}{3}, -0.172\right), \left(\frac{19\pi}{12}, -0.172\right), \left(\frac{11\pi}{6}, -0.172\right)$
- 9 a** $\frac{\pi}{2}, \pi$ **b** $0, \frac{5\pi}{3}$ **c** $0, \frac{3\pi}{2}$ **d** $\frac{\pi}{3}, \frac{\pi}{2}, \frac{5\pi}{6}, \pi, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{11\pi}{6}$ **10 a** 102.4°, 195.7° **b** 7.3°, 34.0°, 127.3°, 154.0°, 247.3°, 274.0°

Chapter 7 Review Exercise

- 1** $\pm \frac{\sqrt{3}}{2}$ **2 b i** $\frac{120}{169}$ **ii** $\frac{119}{169}$ **6** $\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{5\pi}{6}$ **7 a** $5 \cos(\theta - 0.644)$ **b** $\theta = 0.644$ **8** 0.905 **9** $\frac{1 - \sqrt{3}}{2\sqrt{2}}$ **10** $-\frac{13}{85}$ **11** $\frac{2}{\sqrt{2} - \sqrt{2}}$
- 12** $-\frac{35}{12}$ **13** $\sqrt{112} \sin(\theta - 3.86)$ **14** 11.25°, 22.5°, 56.25°, 67.5° **17 b** $\alpha = 0.464, k = -0.559, p = 0.5$ **c** $A = 1.059, \theta = 1.80$

Chapter 8 Exercise 1

- 1** $f'(x) = 5$ **2** $f'(x) = 8$ **3** $f'(x) = -2$ **4** $f'(x) = 2x$ **5** $f'(x) = 3x^2$ **6** $f'(x) = 4x^3$ **7** $f'(x) = 4x$ **8** $f'(x) = 10x$ **9** $f'(x) = 12x^2$
- 10** $f'(x) = 0$ **11** $f'(x) = \frac{-3}{x^2}$ **12** $f'(x) = 2x$ **13** $f'(x) = -3$ **14** $f'(x) = 2x - 4$ **15** $f'(x) = 2 + \frac{1}{x^2}$

Chapter 8 Exercise 2

- 1** $f'(x) = 18x$ **2** $f'(x) = 30x^2$ **3** $f'(x) = 24x^3$ **4** $f'(x) = -15x^4$ **5** $f'(x) = 0$ **6** $f'(x) = 7$ **7** $f'(x) = 11$ **8** $f'(x) = 8$ **9** $f'(x) = -\frac{8}{x^3}$
- 10** $f'(x) = \frac{5}{2\sqrt{x}}$ **11** $f'(x) = 2x + 5$ **12** $f'(x) = -\frac{25}{2\sqrt{x^3}}$ **13** $\frac{dy}{dx} = 3x^2 + 10x - 7$ **14** $\frac{dy}{dx} = 12x + \frac{2}{x^2}$ **15** $\frac{dy}{dx} = \frac{1}{4\sqrt[4]{x^3}}$ **16** $\frac{dy}{dx} = \frac{5\sqrt[3]{x^2}}{3}$
- 17** $\frac{dy}{dx} = \frac{4}{3} + \frac{4}{x^2}$ **18** $\frac{dy}{dx} = \frac{27}{10}(\sqrt{x^7} - \sqrt{x})$ **19** $f'(3) = 2$ **20** $g'(6) = -\frac{10}{9}$ **21** $\frac{dy}{dx} = 6$ **22** $\frac{dy}{dx} = \frac{3081}{128}$ **23** (5, 7)
- 24** $\left(-\frac{1}{2}, \frac{199}{24}\right)$ and $\left(5, -\frac{217}{6}\right)$

Chapter 8 Exercise 3

- 1 a** Tangent $y = 6x - 3$, Normal $y = -\frac{1}{6}x + \frac{19}{6}$ **b** Tangent $y = x - 4$, Normal $y = -x$ **c** Tangent $y = -4x - 3$, Normal $y = \frac{1}{4}x + \frac{5}{4}$
- 1 d** Tangent $y = \frac{1}{6}x + \frac{3}{2}$, Normal $y = -6x + 57$ **e** Tangent $y = -8x + 12$, Normal $y = \frac{1}{8}x + \frac{31}{8}$ **f** Tangent $y = 18x + 47$, Normal $y = -\frac{1}{18}x - \frac{43}{6}$
- 2** At A, $y = 4x - 4$, At B, $y = 3x - 3$ **3** $y = \frac{1}{3}x + \frac{4}{3}$ **4** Q(-3, -20) **5** $y = 2x + 11$ **6** $\left(\frac{1}{4}, -\frac{9}{4}\right)$ **7** $y = -7x - \frac{28}{3}$ and $y = 19x - 57$
- 8** $y = 3x - \frac{38}{9}$ **9** $y = 5x - 15$ and $y = -5x - 10, y = -5x + 15$ and $y = 5x + 10$ **10** $\frac{120}{7}$

Chapter 8 Exercise 4

- 1 a** (4, -13) Minimum Turning Point **b** (-2, 23) Maximum Turning Point and (2, -9) Minimum Turning Point **c** (0, 0) Minimum Turning Point
- 1 d** $\left(-\frac{1}{3}, -\frac{25}{3}\right)$ Minimum Turning Point **e** $\left(-\frac{1}{2}, -4\right)$ Maximum Turning Point and $\left(\frac{1}{2}, 4\right)$ Minimum Turning Point

- 2 a** $(-2, -13)$ Minimum Turning Point **b** $(-1, 25)$ Maximum Turning Point **c** $(\frac{4}{3}, \frac{256}{27})$ Maximum Turning Point and $(4, 0)$ Minimum Turning Point
2 d $(1, 10)$ Maximum Turning Point and $(2, 9)$ Minimum Turning Point **e** $(0, 0)$ Rising Point of Inflexion
3 a $(1, -\frac{8}{3})$ Maximum Turning Point and $(3, -4)$ Minimum Turning Point **b** $(\frac{5}{2}, 0)$ Minimum Turning Point **c** $(-\frac{1}{2}, -12)$ Maximum Turning Point
3 d $(0, 0)$ Minimum Turning Point **e** $(-1.57, 12.5)$ Maximum Turning Point and $(0, 2)$ Minimum Turning Point **4** 2

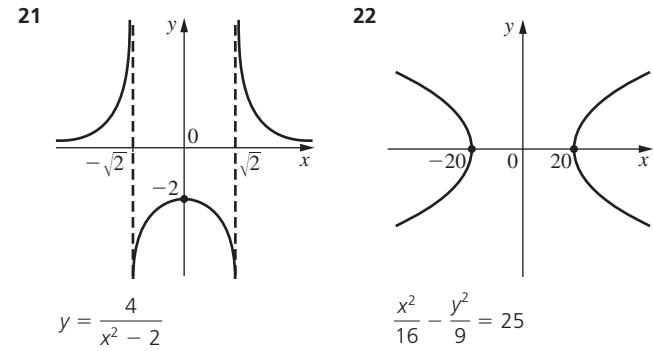
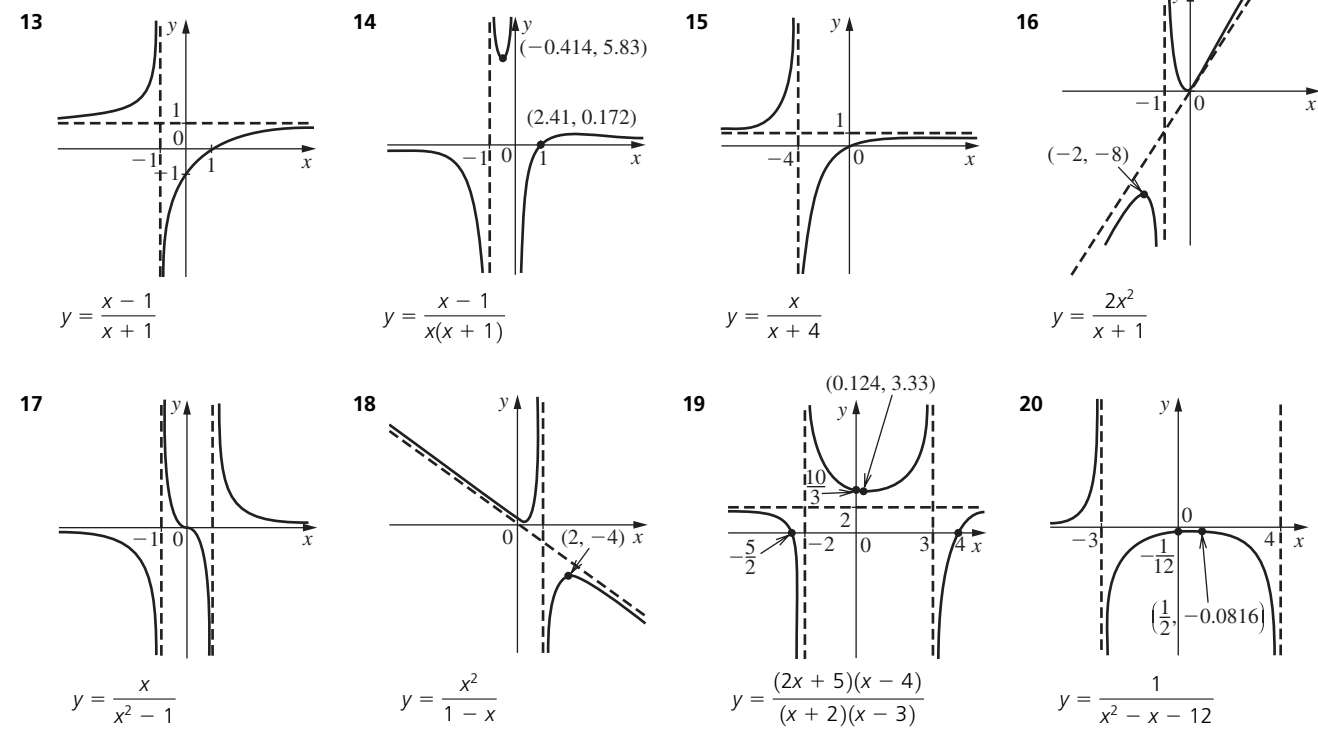
Chapter 8 Exercise 5

- 1** $(-2, \frac{224}{3})$ Non-Stationary, $(0,0)$ Stationary, $(2, -\frac{224}{3})$ Non-Stationary **2** $(-1, 15)$ Non-Stationary **3 a** None **b** None
3 c None Quadratic functions have no points of inflexion **4 a** $(0,0)$ Stationary **b** $(1, -3)$ Non-Stationary **c** $(-2, -35)$ Non-Stationary
4 d $x = -\frac{b}{3a}, y = \frac{8b^3}{27a^2} - \frac{bc}{3a} + d$ There is always one point of inflexion for cubic functions
5 $(-1, 3)$ Non-Stationary and $(1, 3)$ Non-Stationary **6** $(\frac{1}{6}, 4.11)$ Non-Stationary and $(-1, -9)$ Non-Stationary
7 $(0,0)$ Stationary **8** No points of inflexion, $(0, -3)$ is a minimum turning point. **9** $y = -21x + 36$ **10** $\sqrt{1312} \approx 36.2$

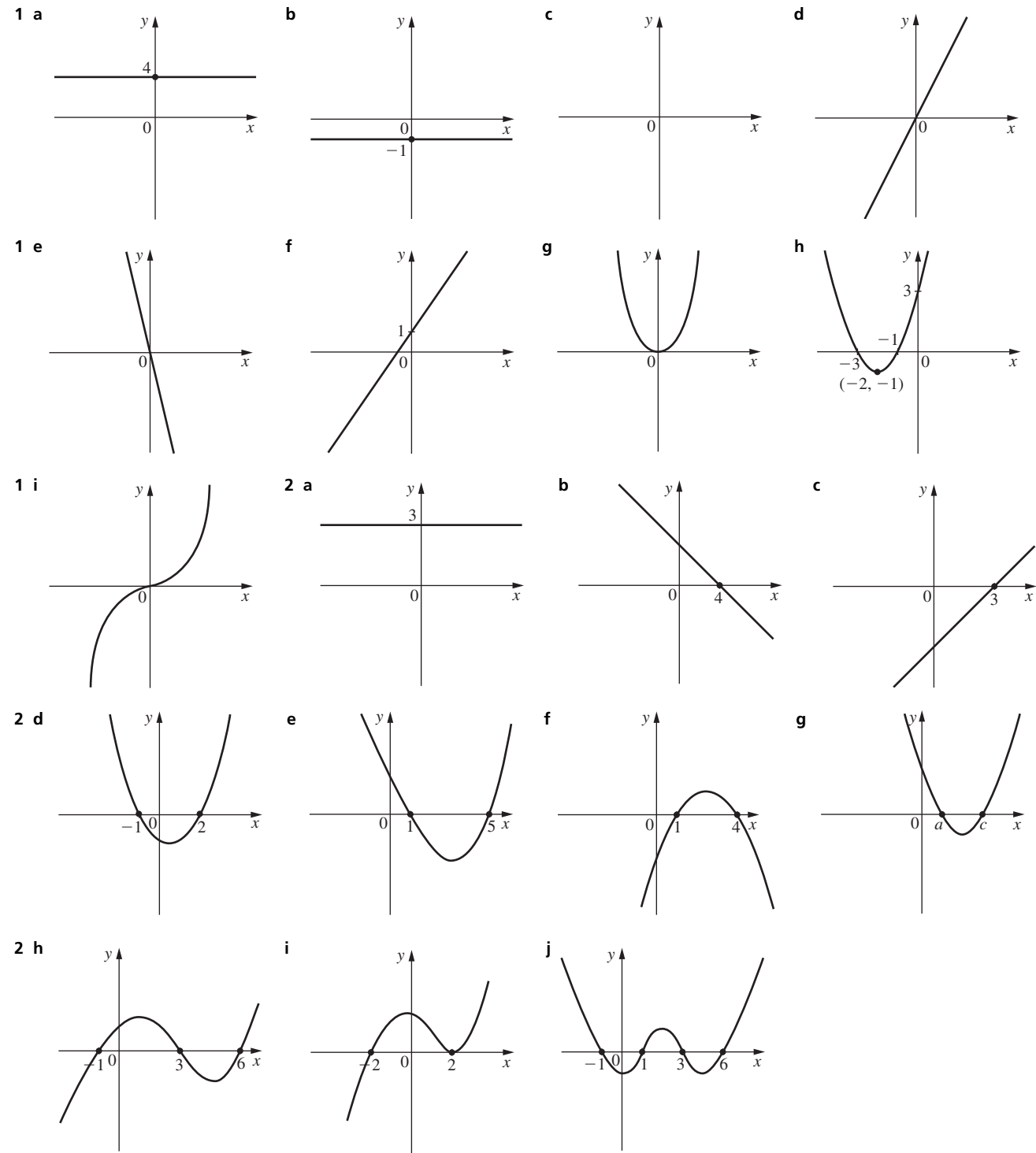
Chapter 8 Exercise 6

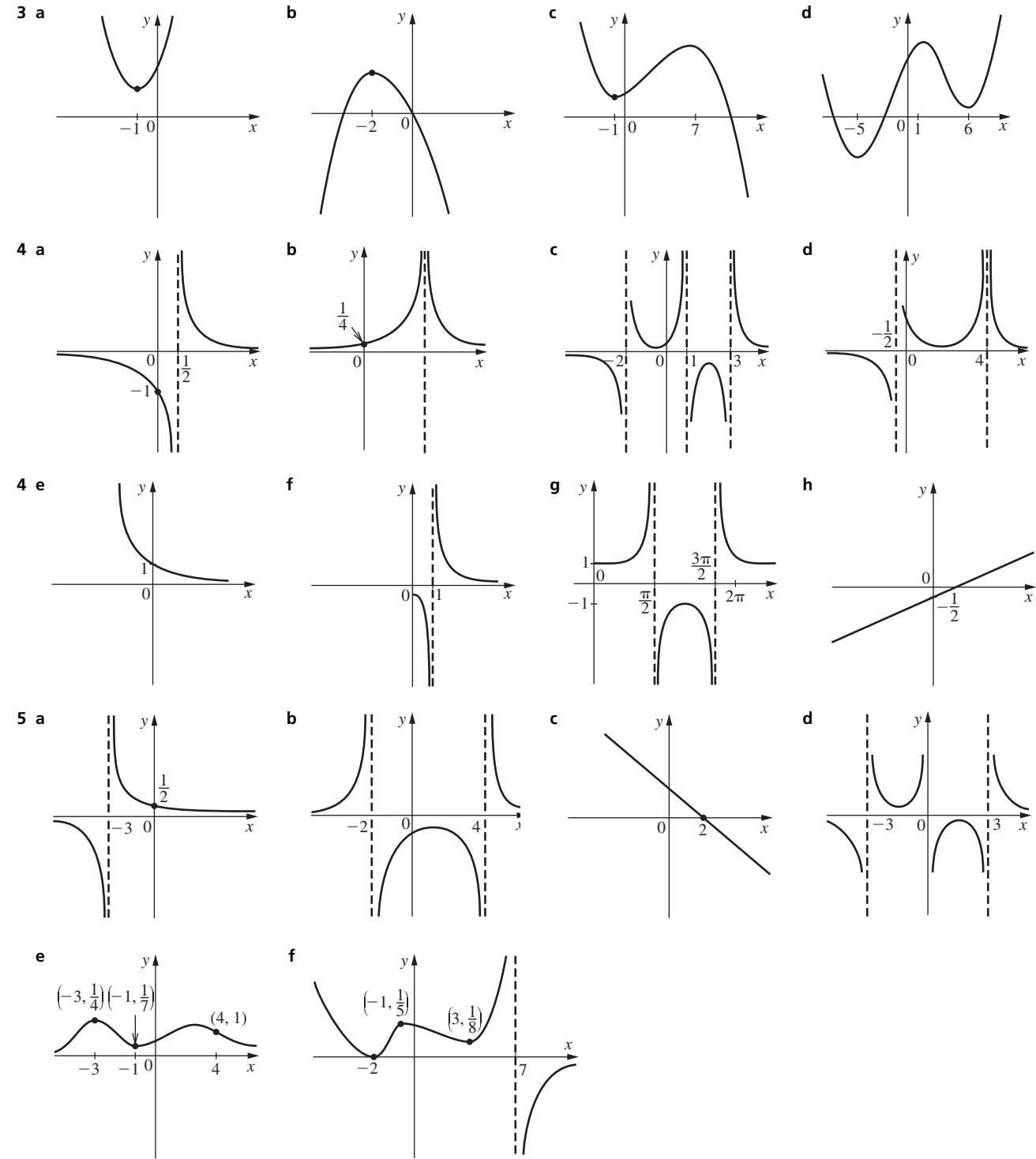
- 1** Vertical: $x = 0$, Horizontal $y = 0$ **2** Vertical: $x = 3$, Horizontal $y = 1$ **3** Vertical: $x = 0$, Oblique $y = x$
4 Vertical: $x = 2$, Horizontal $y = 1$ **5** Vertical: $x = -2$, Oblique $y = x - 2$ **6** Vertical: $x = 3$, Oblique $y = 2x + 9$
7 Vertical: None, Oblique $y = x - 2$ **8** Vertical: None, Oblique $y = x$ **9** Vertical: $x = -1$ and $x = 4$, Horizontal $y = 0$
10 Vertical: $x = -1$ and $x = 1$, Horizontal $y = 1$ **11** Vertical: $x = -3$ and $x = 3$, Horizontal $y = 3$
12 Vertical: $x = -2$ and $x = 3$, Oblique $y = 4x + 4$

Graphs of the following functions, including asymptotes, stationary points and intercepts:



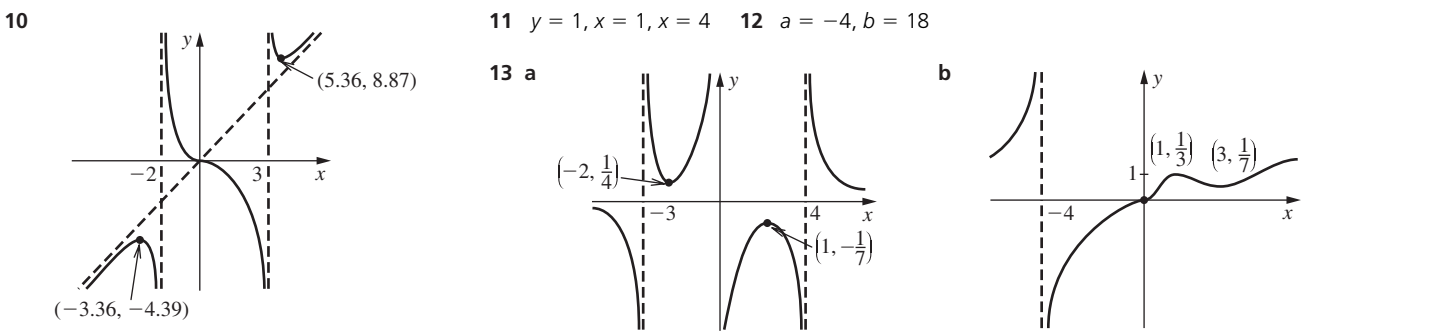
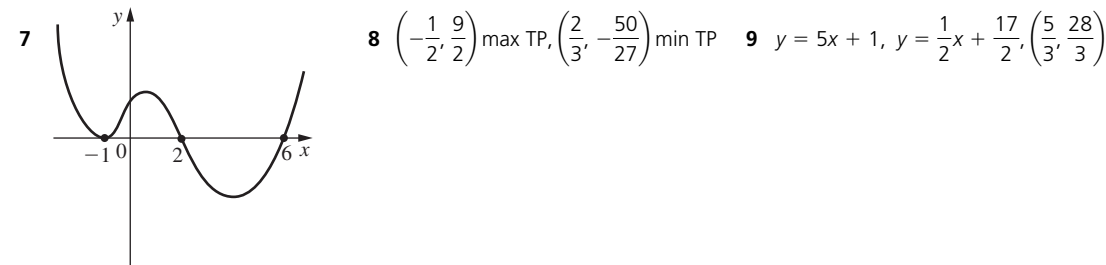
Chapter 8 Exercise 7





Chapter 8 Review Exercise

1 $f'(x) = 3x^2 - 4$ **2** $\frac{1}{16}$ **3** $\frac{dy}{dx} = \frac{3}{4}x^{-\frac{1}{2}} - \frac{17}{4}x^{\frac{15}{2}}$ **4** $x > 4$ **5** $h(x) = 45x^2 + 60x + 19, h'(x) = 90x + 60$ **6** $y = -6x + 16$



Chapter 9 Exercise 1

1 $\frac{dy}{dx} = \sec^2 x$ **2** $\frac{dy}{dx} = \cos x - \operatorname{cosec} x \cot x$ **3** $\frac{dy}{dx} = \cos x + 12x$ **4** $\frac{dy}{dx} = -5 \sin x$ **5** $\frac{dy}{dx} = -7 \operatorname{cosec}^2 x$ **6** $\frac{dy}{dx} = -3 \sec x \tan x$

7 $\frac{dy}{dx} = 18x + 4 \sin x$ **8** $\frac{dy}{dx} = 7 - 5 \cos x - \sec x \tan x$

Chapter 9 Exercise 2

1 $f'(x) = 2(x + 4)$ **2** $f'(x) = 4(2x + 3)$ **3** $f'(x) = 6(3x - 4)$ **4** $f'(x) = 20(5x - 4)^3$ **5** $f'(x) = -3(5 - x)^2$ **6** $f'(x) = -8(7 - 2x)^3$

7 $\frac{dy}{dx} = -20(9 - 4x)^4$ **8** $\frac{dy}{dx} = 48(2x + 3)^5$ **9** $\frac{dy}{dx} = \frac{3}{2}(3x + 8)^{-\frac{1}{2}}$ **10** $\frac{dy}{dx} = \frac{10}{3}(2x - 9)^{\frac{2}{3}}$ **11** $\frac{dy}{dx} = 2(6x - 5)^{-\frac{2}{3}}$

12 $\frac{dy}{dx} = -\frac{3}{2}(3x - 2)^{-\frac{3}{2}}$ **13** $f'(x) = -20(5x - 4)^{-2}$ **14** $f'(x) = 56(3 - 8x)^{-2}$ **15** $\frac{dP}{dk} = 18(4 - 3k)^{-3}$ **16** $\frac{dN}{dp} = \frac{75}{2}(8 - 5p)^{-\frac{5}{2}}$

17 $\frac{dy}{dx} = 4 \cos 4x$ **18** $\frac{dy}{dx} = -3 \sin 3x$ **19** $\frac{dy}{dx} = -\frac{1}{2} \cos \frac{1}{2}x$ **20** $\frac{dy}{dx} = 6 \sec^2 6x$ **21** $\frac{dy}{dx} = 9 \sec 9x \tan 9x$ **22** $\frac{dy}{dx} = 6 - 3 \operatorname{cosec}^2 3x$

23 $\frac{dy}{dx} = -2 \operatorname{cosec} 2x \cot 2x + 12(3x + 2)^3$ **24** $\frac{dy}{dx} = 5 \cos 5x + 30(3x + 4)^{-\frac{1}{2}}$ **25** $\frac{dy}{dx} = 3 \sin^2 x \cos x$ **26** $\frac{dy}{dx} = 8 \tan 4x \sec^2 4x$

27 $\frac{dy}{dx} = 12x^3 + 3 \cos^2 x \sin x$ **28** $\frac{dy}{dx} = -30(3x - 4)^{-6} - 2 \sec^2 2x \tan 2x$ **29** $\frac{dy}{dx} = -3 \sin\left(3x - \frac{\pi}{4}\right)$ **30** $\frac{dy}{dx} = (x + 1)^{-\frac{1}{2}} \sec^2(\sqrt{x + 1})$

Chapter 9 Exercise 3

1 $f'(x) = 3e^{3x}$ **2** $f'(x) = 7e^{7x}$ **3** $f'(x) = -4e^{4x}$ **4** $f'(x) = -10e^{-5x}$ **5** $f'(x) = 54e^{-9x}$ **6** $f'(x) = 2xe^{x^2}$ **7** $f'(x) = 2e^{2x+3}$ **8** $f'(x) = \frac{1}{x}$

9 $f'(x) = \frac{1}{x}$ **10** $f'(x) = -\frac{2}{x}$ **11** $f'(x) = \frac{2x}{x^2 + 2}$ **12** $\frac{dy}{dx} = \ln 4 \cdot 4^x$ **13** $\frac{dy}{dx} = \ln 10 \cdot 10^x$ **14** $\frac{dy}{dx} = 6 \ln 5 \cdot 5^x$ **15** $\frac{dy}{dx} = 3e^{3x} - \ln 3 \cdot 3^x$

16 $\frac{dy}{dx} = \frac{1}{x} - \ln 2 \cdot 2^x$ **17** $\frac{dy}{dx} = \frac{1}{x \ln 2}$ **18** $\frac{dy}{dx} = \frac{1}{x \ln 8}$ **19** $\frac{dy}{dx} = \ln 4 \cdot 4^x - \frac{1}{x \ln 5}$ **20** $\frac{dy}{dx} = 4e^{4x} - 2 \cos 2x + \frac{1}{x}$ **21** $\frac{dy}{dx} = -\tan x$

22 $\frac{dy}{dx} = \operatorname{cosec} x \sec x$ **23** $\frac{dy}{dx} = \frac{1}{x} \sec^2(\ln x)$

Chapter 9 Exercise 4

1 $\frac{dy}{dx} = x(2 \sin x + x \cos x)$ **2** $\frac{dy}{dx} = x^2(3 \cos x - x \sin x)$ **3** $\frac{dy}{dx} = 3xe^x(2 + x)$ **4** $\frac{dy}{dx} = e^{3x}(3 \sin x + \cos x)$ **5** $\frac{dy}{dx} = \frac{\sin x}{x} + \ln x \cos x$

6 $\frac{dy}{dx} = \cos^2 x - \sin^2 x$ **7** $\frac{dy}{dx} = 3 \cos 3x \cos 2x - 2 \sin 3x \sin 2x$ **8** $\frac{dy}{dx} = 2x(x - 1)(2x - 1)$ **9** $\frac{dy}{dx} = x^2(x - 2)^3(3 + 4x)$

10 $\frac{dy}{dx} = 6x^2(3x + 2)(5x + 2)$ **11** $\frac{dy}{dx} = (2x + 1)^2(8x - 11)$ **12** $\frac{dy}{dx} = 2(x + 5)(3x - 2)^3(9x + 28)$ **13** $\frac{dy}{dx} = 6(5 - 2x)^2(3x + 4)(1 - 5x)$

14 $\frac{dy}{dx} = (3x + 4)^2[9 \sin x + (3x + 4) \cos x]$ **15** $\frac{dy}{dx} = 5^x(\ln 5 \cdot \cos x - \sin x)$ **16** $\frac{dy}{dx} = x^2\left(3 \log_6 x + \frac{1}{\ln 6}\right)$ **17** $\frac{dy}{dx} = e^{4x} \sec 3x(4 + 3 \tan 3x)$

18 $\frac{dy}{dx} = 3(2x + 1)^2 \operatorname{cosec} 3x[2 - (2x - 1) \cot 3x]$ **19** $\frac{dy}{dx} = 4^x\left(\ln 4 \log_8 x + \frac{1}{x \ln 8}\right)$ **20** $\frac{dy}{dx} = \ln(2x + 3) + \frac{2x}{2x + 3}$

21 $\frac{dy}{dx} = 8x\left[\ln(x^2 + 2x + 5) + \frac{x(x + 1)}{x^2 + 2x + 5}\right]$ **22** $\frac{dy}{dx} = e^{3x} \sec\left(2x - \frac{\pi}{4}\right)\left[3 + 2 \tan\left(2x - \frac{\pi}{4}\right)\right]$

23 $\frac{dy}{dx} = -3x^{-5}\left(4 \tan\left(3x + \frac{\pi}{2}\right) - 3x \sec^2\left(3x + \frac{\pi}{2}\right)\right)$ **24** $\frac{dy}{dx} = x(2 \ln x + 1) \sin x + x \ln x \cos x$ **25** $\frac{dy}{dx} = e^{3x}(x + 2)[(3x + 8) \tan x + (x + 2) \sec^2 x]$

Chapter 9 Exercise 5

1 $f'(x) = \frac{e^x(\cos x + \sin x)}{\cos^2 x}$ 2 $f'(x) = \frac{6x(x+6)}{(x+3)^2}$ 3 $f'(x) = \frac{7 \tan x - 7x \sec^2 x}{\tan^2 x}$ 4 $f'(x) = \frac{1 - \ln x}{4x^2}$ 5 $f'(x) = \frac{e^x(x-5)}{(x-4)^2}$

6 $f'(x) = \frac{-6}{(x-3)^2}$ 7 $f'(x) = \frac{-3(x+12)}{2x^3\sqrt{x+9}}$ 8 $f'(x) = \frac{4^x(2x \ln 4 - 1)}{2x^{\frac{3}{2}}}$ 9 $f'(x) = \frac{x-2}{(x-1)^{\frac{3}{2}}}$ 10 $\frac{dy}{dx} = \frac{e^{3x}(3x-2)}{9x^3}$

11 $\frac{dy}{dx} = \frac{\frac{1}{x \ln 6}(x+6) - \log_6 x}{(x+6)^2}$ 12 $\frac{dy}{dx} = \frac{\frac{1}{x} \ln(x-4) - \frac{1}{x-4} \ln x}{(\ln(x-4))^2}$ 13 $\frac{dy}{dx} = \frac{-2}{(e^x - e^{-x})^2}$ 14 $\frac{dy}{dx} = \frac{2(\cos 2x - 3 \sin 2x)}{e^{6x}}$

15 $\frac{dy}{dx} = \frac{12(3x-2)^4(4x+19)}{(2x+3)^4}$ 16 $\frac{dy}{dx} = \frac{(x+1) \sin x + x \cos x}{e^x}$ 17 $\frac{dy}{dx} = \frac{xe^{3x}(3x^2+15x+10)}{(x+5)^2}$ 18 $\frac{dy}{dx} = \frac{\tan\left(x + \frac{\pi}{4}\right) - 2}{e^{2x} \cos\left(x + \frac{\pi}{4}\right)}$

19 $\frac{dy}{dx} = \frac{-2 \operatorname{cosec}^2 x \left(2x - \frac{\pi}{3}\right) \ln(3x-1) - \frac{3}{3x+1} \cot\left(2x - \frac{\pi}{3}\right)}{(\ln(3x+1))^2}$

Chapter 9 Exercise 6

1 $\frac{dy}{dx} = \frac{-3x^2 - y}{x}$ 2 $\frac{dy}{dx} = \frac{-4x}{y}$ 3 $\frac{dy}{dx} = \frac{1}{6\sqrt{xy^2}}$ 4 $\frac{dy}{dx} = \frac{1 - xy - 2x}{x(x+3)}$ 5 $\frac{dy}{dx} = \frac{y}{2y-x}$ 6 $\frac{dy}{dx} = \frac{2(x-y)}{e^y + 2x - 2y}$

7 $\frac{dy}{dx} = \frac{-(3 \cos 3x + 2e^{2x}y^3)}{3e^{2x}y^2}$ 8 $\frac{dy}{dx} = \frac{-\sin(x+y)}{1 + \sin(x+y)}$ 9 $\frac{dy}{dx} = \frac{4x^3}{1 + \ln y}$ 10 $\frac{dy}{dx} = \frac{-3(x+y)^2}{3(x+y)^2 - e^y}$ 11 $\frac{dy}{dx} = \frac{y^2(8 - e^x) - 4y(x+y)^3}{(x+y)^3(3y-x)}$

12 $\frac{dy}{dx} = \frac{x}{2+3y}$ $\frac{d^2y}{dx^2} = \frac{(2+3y)^2 - 3x(2+3y) - x^2}{(2+3y)^3}$ 13 $\frac{dy}{dx} = \frac{\cos x - 4y}{4x+1}$ $\frac{d^2y}{dx^2} = \frac{(4x+1)\left(-\sin x - 4\frac{dy}{dx}\right) - 4(\cos x - 4y)}{(4x+1)^2}$

14 $\frac{dy}{dx} = -\frac{1}{x}$ $\frac{d^2y}{dx^2} = \frac{1}{x^2}$ 15 $y = \frac{3}{2}x - \frac{5}{2}, y = \frac{1}{2}x + \frac{5}{2}$ 16 -19. Answers given

Chapter 9 Exercise 7

1 $\frac{dy}{dx} = \frac{1}{\sqrt{25-x^2}}$ 2 $\frac{dy}{dx} = \frac{-1}{\sqrt{64-x^2}}$ 3 $\frac{dy}{dx} = \frac{10}{100+x^2}$ 4 $\frac{dy}{dx} = \frac{2}{\sqrt{9-4x^2}}$ 5 $\frac{dy}{dx} = \frac{-3}{\sqrt{1-9x^2}}$ 6 $\frac{dy}{dx} = \frac{2e^x}{4+e^{2x}}$

7 $\frac{dy}{dx} = \frac{-1}{2\sqrt{-(x+4)(x+3)}}$ 8 $\frac{dy}{dx} = \frac{1}{2x^2-2x+1}$ 9 $\frac{dy}{dx} = \frac{1}{x\sqrt{1-(\ln 5x)^2}}$

Chapter 9 Exercise 8

1 $f'(x) = 2x - 5$ 2 $\frac{dy}{dx} = 6(2x-7)^2$ 3 $f'(x) = -8 \sin 8x - \frac{3}{2}x^{-\frac{1}{2}}$ 4 $\frac{dy}{dx} = \sec x \tan x - 5e^{5x}$ 5 $f'(x) = x^2e^{-4x}(3-4x)$

6 $\frac{dy}{dx} = x(2 \ln x + 1)$ 7 $f'(x) = \frac{3 \cos 3x - \sin 3x}{e^x}$ 8 $\frac{dy}{dx} = \frac{\sin x - 4x \sin x + x \cos x}{e^{4x}}$ 9 $f'(x) = 3^x(\ln 3 \cdot \sin x + \cos x)$

10 $\frac{dy}{dx} = \frac{\frac{x-4}{x \ln 2} - 3 \log_2 x}{(x-4)^4}$ 11 $\frac{dy}{dx} = \frac{x \ln x(x+18) + x(x+9)}{(x+9)^2}$ 12 $\frac{dy}{dx} = -6(\sin 2x \sin 4x + 2 \cos 2x \cos 4x)$ 13 $\frac{dy}{dx} = \frac{12}{\sqrt{1-4x^2}}$

14 $f'(x) = \frac{-\left(\frac{x}{\sqrt{1-x^2}} + 2 \cos^{-1} x\right)}{3x^3}$ 15 $\frac{dy}{dx} = \sin x(\ln x + 1) + x \ln x \cos x$ 16 $\frac{dy}{dx} = \frac{-(\sec x \operatorname{cosec} x + \ln(\cot x))}{e^x}$ 17 $f'(2) = -\frac{20}{3}$

18 $f'(4) = \frac{-\pi+2}{64}$ 19 $\frac{dy}{dx} = \frac{-y(2x+e^xy)}{x^2+2e^xy}$ 20 $\frac{dy}{dx} = \frac{y(4x^3y^2-\cos x)}{(\sin x-3x^4y^2)}$

Chapter 9 Exercise 9

1 $\frac{3}{4}$ 2 0 3 $\frac{4 \ln 4 - 3}{e^2}$ 4 $y = -\frac{7}{5}x + \frac{17}{5}, y = -\frac{3}{5}x + \frac{13}{5}$ 6 $\frac{2-2\pi}{e^\pi}$ 7 (0,0) and $\left(2, \frac{4}{e^2}\right)$ 8 (0, 0) and $\left(e^{-\frac{1}{2}}, -\frac{2}{e}\right)$

9 (0, 0) rising point of inflexion and $\left(3, \frac{27}{e^3}\right)$ max TP

Chapter 9 Review Exercise

1 a $60(3x-2)^3$ b $14x(3-2x^2)^{-\frac{3}{2}}$ c $6-3 \tan 3t \sec 3t$ d $48e^{8x}$ e $\frac{1}{x} - 3^x \ln 3$ 2 a $4e^{4x} \sin 3x + 3e^{4x} \cos 3x$ b $\frac{1}{x} + \cot x$

2 c $\frac{e^{5x}}{\sqrt{x+4}}\left(5 - \frac{1}{2(x+4)}\right)$ d $\frac{3}{3x+4} - \frac{2}{2x-1}$ e $\frac{1}{\ln 10}\left(2 - 3 \tan 3x - \frac{2}{x+4}\right)$ 3 a $\frac{6xy}{8y-3x^2}$ b $\frac{3x^2}{\ln x} - \frac{y}{x \ln x}$

4 $\frac{2 \sin x + 4x \cos x - 2x \sin x - x^2(\cos x + \sin x)}{e^x} - \frac{2x \sin x - x^2 \cos x}{e^{2x}} + y$ 5 $\frac{-2(1 + \csc x \cot x + \cot^2 x)}{1 + \left(\frac{1 + \cos x}{\sin x}\right)^2}$ 6 $-\frac{16\sqrt{x} - 4\pi\sqrt{x}}{\pi^2}$ 7 $-\frac{3}{4}$

8 $-\frac{1}{3}$ 9 $\frac{4}{3}$ 10 $x = 0$ 11 2.533 12 $y = \frac{x}{4} - \frac{5}{2}$ 13 $-2(2x-1)^{-2}$ 14 a 6 sec 2t tan 2t + 5 b i 3 + 5\pi ii 5 15 a -1 b $\frac{4}{5}$

Chapter 10 Exercise 1

1 $v = \sqrt{500}ms^{-1}$ 2 All three sides are $\sqrt{50}cm$ 3 6.83cm 4 5.62cm, 4.22cm, 16.9cm 5 4.16cm, 6.56cm 6 3.66cm² 7 78.5 8 b $\frac{4r}{3}$

9 $x = y = 75cm$. It is a maximum value. 10 b $V = 60\left(\frac{450x - 60x^3}{13}\right)$ 10 c $x = 1.58cm$. It is a maximum value.

11 $t = 6.29$ seconds, $t = 12.6$ seconds 12 11100 13 12.5 Yen, 24.1Kmh⁻¹ 14 20 15 a $\frac{7-x}{7} = \frac{y}{24}$ b $A = \frac{168x - 24x^2}{7}$ c 42cm²

Chapter 10 Exercise 2

1 128\pi cm^2s^{-1} 2 $\frac{15}{32\pi}$ 3 $\frac{2}{h}$ 4 1640N₀ 5 216cm³s⁻¹ 6 0.954ms⁻¹ 7 1.49cm²s⁻¹ 8 0.283cms⁻¹ 9 $-\frac{5}{3}$

10 a $(a\theta - a \sin \theta, a - a \cos \theta)$ 11 100cm³s⁻¹ 13 a $\frac{2}{9}cms^{-1}$ b $\frac{4\pi}{3}cm^2s^{-1}$

Chapter 10 Exercise 3

1 $v = 3\frac{1}{4}ms^{-1}$ a = -3ms⁻² $t = \frac{2\sqrt{3}}{3}$ seconds 2 a 8ms⁻¹ b 16 - 12t c -20ms⁻² 3 a -4(1 - 2t) b $t = \frac{1}{2}$ second c 0ms⁻²

4 a $v = \frac{t^2 \cos t - t \cos t - \sin t}{(t-1)^2}$ b -1.74ms⁻¹ c $a = \frac{t^3 \sin t + 2t^2 \sin t - 2t \cos t + 2 \cos t - t \sin t + 2 \sin t}{(t-1)^3}$ 7 Period = \pi

8 a $s = \frac{e^{2t}(t^2 - 2t - 1)}{(t^2 - 1)^2}$ b $v = \frac{-e^4}{9}$ c $a = \frac{2e^{2t}(t^4 - 6t^3 + 4t^2 + 6t - 1)}{(t^2 - 1)^3}$ d $a = \frac{-10e^4}{9}$ 10 a $v = 3k \cos(kt + c)$

10 b $t = \frac{\pi - 2c}{2k}$ c $a = 0.0213ms^{-2}$ 11 a $k = \frac{1}{100} \ln 300$ b. $v = e^{\frac{\ln 300}{100}t^2}\left(1 + \frac{\ln 300}{50}t^2\right)$ c $a = \frac{\ln 300}{50}e^{\frac{\ln 300}{100}t^2}\left(3t + \frac{\ln 300}{50}t^2\right)$ d 0.629

12 a Min is (2,1.39) b $v = \frac{2}{t} - \frac{1}{t-1}$ $t \geq 2$ c $a = -\frac{2}{t^2} - \frac{1}{(t-1)^2}$ $t \geq 2$ d $v = \frac{4}{45}ms^{-1}$ $a = -\frac{31}{4050}ms^{-2}$

13 a $v = \frac{2t-s}{t+s}$ b $a = \frac{3(s^2+2st-2t^2)}{(t+s)^3}$ c $s = t(-1 \mp \sqrt{3})$

Chapter 10 Review Exercise

1 240 kmh⁻¹ 2 a 13 ms⁻² b $t = 4.70$ seconds and $t = 1.05$ seconds

2 c Maximum velocity is 0.541 ms⁻¹ Minimum velocity is -1.35 ms⁻¹ 3 $\frac{1}{2\pi}cms^{-1}$ 4 a = 26.2 cm b = 22.7 cm 5 -1.6

6 a $A = 2\pi rh + \pi r^2$ b 4.30 cm c 1120 cm² 7 a 6t sin 5t + 15t² cos 5t b 0.457 hours c 0.304 hours 8 a $(\pi - 2x) \sin x$

8 b 1.12 units² 9 $\frac{\pi}{3}cm^2 s^{-1}$ 10 $-\frac{56}{27}ms^{-2}$ 11 a 6.28 hours. b 6.28 hours. 12 $\frac{704}{3}cm^3 s^{-1}$ 13 a $BC = 2(x - h \csc \theta)$

13 b $2xh + h^2(\cot \theta - 2 \csc \theta)$ c $h(2x - h\sqrt{3})$ 14 a $t = 2$ minutes b $\frac{112t^3}{(8+t^4)^2}cm \min^{-1}$ c 27.7 cm² min⁻¹ d 1.48 minutes

Chapter 11 Exercise 1

1 a 1×3 b 2×3 c 3×3 d 4×1 2 Week 1 $\begin{pmatrix} 3 & 1 \\ 2 & 2 \\ 4 & 4 \end{pmatrix}$ Week 2 $\begin{pmatrix} 1 & 2 \\ 4 & 1 \\ 0 & 1 \end{pmatrix}$ Week 3 $\begin{pmatrix} 4 & 2 \\ 1 & 0 \\ 1 & 1 \end{pmatrix}$

The operation is addition.

	Magazines	Newspapers
Alan	8	5
Bill	7	3
Colin	5	6

3 a $\begin{pmatrix} 8 & 4 & 12 \\ 20 & -8 & 12 \\ 28 & -16 & 4 \end{pmatrix}$ **b** $\begin{pmatrix} -18 & -24 \\ 6 & -12 \\ 18 & 24 \end{pmatrix}$ **c** $\begin{pmatrix} 3k & 6k \\ -4k & -k \\ 12k & 4k \end{pmatrix}$ **d** $\begin{pmatrix} 3(k-1) & 2(k-1) \\ 1-k & 0 \end{pmatrix}$ **4 a** $k = 6$ **b** $k = 0, 1$ **c** $k = 1$ **d** $k = 0, 3$

4 e $k = \frac{1}{2}$ **f** $k = 5$ **g** $x = 0, y = 11$ **5 a** not possible **b** $\begin{pmatrix} 10 & -2 \\ 0 & 4 \end{pmatrix}$ **c** not possible **d** $\begin{pmatrix} -11 & 5 \\ 6 & 0 \\ 8 & 1 \end{pmatrix}$ **e** $\begin{pmatrix} 7 & 1 \\ -1 & 3 \end{pmatrix}$ **f** $\begin{pmatrix} 9 & -2 \\ 0 & 3 \end{pmatrix}$

5 g $\begin{pmatrix} 18 & 5 \\ -8 & -15 \\ -4 & 37 \end{pmatrix}$ **h** $\begin{pmatrix} -15 & 1 \\ 1 & -5 \end{pmatrix}$ **6 a** $\begin{pmatrix} -24 & 11 \\ -6 & 29 \end{pmatrix}$ **b** (-26) **c** $\begin{pmatrix} 11 & 29 \\ 6 & 22 \\ 12 & -7 \end{pmatrix}$ **d** $\begin{pmatrix} 29 & 39 & -11 \\ 11 & 41 & -9 \\ 2 & -29 & 2 \end{pmatrix}$ **e** $\begin{pmatrix} 6+k^2 & 2+2k \\ k^2-3 & 2k-1 \end{pmatrix}$

6 f $\begin{pmatrix} 7+3k^2-4k \\ -5 \\ 4-k \\ 1+2k-2k^2 \end{pmatrix}$ **7 a** $\begin{pmatrix} 45 & 4 \\ 39 & -18 \end{pmatrix}$ **b** $\begin{pmatrix} 45 & 45k+4 \\ 39 & 39k-18 \end{pmatrix}$ **c** $\begin{pmatrix} 45 & 45k+4 \\ 39 & 39k-18 \end{pmatrix}$ **d** $\begin{pmatrix} 45+39k & 4-18k \\ 39 & -18 \end{pmatrix}$ **e** $\begin{pmatrix} 9 & 9k+12 \\ 27 & 27k-6 \end{pmatrix}$

7 f $\begin{pmatrix} 0 & 0 \\ -11 & 7-11k \end{pmatrix}$ **g** $\begin{pmatrix} -6 & 84-9k \\ -6 & 52-5k \end{pmatrix}$ **h** $\begin{pmatrix} -264 & 168 \\ -99 & 63 \end{pmatrix}$ **8 a** $x = 2, y = 1$ **b** $x = 0, y = 1$ **c** $x = 10, y = 1$ **d** $x = 4, y = -3$

9 $A^2 = \begin{pmatrix} 13 & -4 \\ 12 & -3 \end{pmatrix}$ $A^3 = \begin{pmatrix} 40 & -13 \\ 39 & -12 \end{pmatrix}$ **10 a** $A = \begin{pmatrix} 3 & 2 & 4 \\ 5 & 7 & 2 \end{pmatrix}$ **b** $B = \begin{pmatrix} 1900 \\ 1300 \\ 1100 \end{pmatrix}$ **c** $AB = \begin{pmatrix} 12700 \\ 20800 \end{pmatrix}$

Total number of calories consumed on day 1 and day 2. **d i** (33500). Total calories consumed. **ii** $\begin{pmatrix} 9 \\ 14 \end{pmatrix}$ Total number of people dieting on each day.

iii (4300) Total number of calories consumed by one man, one women and one child. **11 a** $P = \begin{pmatrix} 3 & 4 & 7 \\ 6 & 2 & 6 \\ 10 & 1 & 3 \\ 3 & 9 & 2 \\ 8 & 3 & 3 \end{pmatrix}$ **b** $Q = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$ **c** $\begin{pmatrix} 13 \\ 31 \\ 18 \\ 27 \end{pmatrix}$

12 a $\begin{pmatrix} -1 & 6 \\ -4 & 1 \end{pmatrix}$ **b** $\begin{pmatrix} -7 & 9 \\ -6 & -4 \end{pmatrix}$ **c** $\begin{pmatrix} -6 & 0 \\ 0 & -6 \end{pmatrix}$ **13** $m = 1, n = -6$ **14** $k = 10$ **16** $c = 5$ **17** $c = -2$

18 $PQ = \begin{pmatrix} 4 & 1-3c \\ 5c+4 & c \end{pmatrix}$ $QP = \begin{pmatrix} 5+c & 22 & -15 \\ 2 & 8 & -6 \\ 3+c^2 & 12+2c & -9 \end{pmatrix}$ **19 a** $x = -\frac{8}{5}, y = 10$ **b** $x = -5, y = 0$ **c** $x = \frac{13}{3}, y = 0$ **20** $\begin{pmatrix} a & b \\ 0 & a \end{pmatrix}$

Chapter 11 Exercise 2

1 a $\frac{1}{11} \begin{pmatrix} 5 & 2 \\ -3 & 1 \end{pmatrix}$ **b** $\frac{1}{41} \begin{pmatrix} 10 & -7 \\ 3 & 2 \end{pmatrix}$ **c** $\frac{1}{76} \begin{pmatrix} 1 & -8 \\ 9 & 4 \end{pmatrix}$ **d** $\begin{pmatrix} \frac{11}{137} & -\frac{6}{137} & \frac{29}{274} \\ \frac{10}{137} & \frac{7}{137} & -\frac{11}{274} \\ \frac{24}{137} & \frac{38}{137} & -\frac{1}{274} \end{pmatrix}$ **e** $\begin{pmatrix} \frac{2}{37} & -\frac{25}{333} & -\frac{8}{333} \\ \frac{3}{74} & \frac{1}{37} & \frac{11}{74} \\ \frac{5}{74} & \frac{5}{111} & -\frac{19}{222} \end{pmatrix}$ **f** $\begin{pmatrix} \frac{37}{304} & -\frac{33}{152} & \frac{9}{76} \\ -\frac{1}{38} & \frac{5}{19} & -\frac{1}{19} \\ -\frac{17}{152} & \frac{9}{76} & \frac{1}{38} \end{pmatrix}$

1 g $-\frac{1}{14k} \begin{pmatrix} 1 & -5 \\ -3k & k \end{pmatrix}$ **h** $\frac{1}{k^2+7k} \begin{pmatrix} k+2 & -k \\ 1-2k & 3k \end{pmatrix}$ **2** $X = \begin{pmatrix} 1 & -\frac{3}{14} \\ -1 & \frac{5}{14} \end{pmatrix}$ $Y = \begin{pmatrix} \frac{5}{2} & \frac{1}{2} \\ 7 & 5 \end{pmatrix}$ $Z = \begin{pmatrix} \frac{43}{18} & \frac{31}{18} \\ \frac{19}{18} & -\frac{11}{18} \end{pmatrix}$ **3 a** -30 **b** 44 **c** 78 **d** 190

4 a $\cos^2 \theta + \sin^2 \theta = 1$ **b** $-\sin 2\theta$ **c** -2 **d** $2abc$ **e** $\cos^3 \theta + \sin^3 \theta - 3 \sin^2 \theta + \tan^3 \theta$ **f** $-y^2 + y - 2$ **g** $a^3 - a^2b - ab^2 + b^3$

5 a 4 units^2 **5 b** 4 units^2 **c** 20 units^2 **6 a** Collinear **b** Not collinear **c** Collinear **8** $k = -1.22, 1.08$ **10** $c = -5, 4.5$

11 $y = -3.36, 3.86$ **12** $x = y = 1$ **13 a** $\frac{1}{3}B^{-1}(2A - B)$ **b** $X = I$ **14 b** $\frac{1}{8} \begin{pmatrix} -x-3 & x-1 \\ x+1 & 3-x \end{pmatrix}$

Chapter 11 Exercise 3

1 a $x = \frac{15}{7}, y = -\frac{17}{7}$ **b** $p = -21, q = -14$ **c** $x = \frac{5}{6}, y = \frac{2}{9}$ **d** $x = 7, y = 17$ **2 a** $x = \frac{17}{4}, y = -\frac{11}{20}$ **b** $a = \frac{19}{11}, b = -\frac{9}{11}$

2 c $x = -\frac{5}{7}, y = \frac{22}{7}$ **d** $x = -\frac{16}{5}, y = -\frac{69}{5}$ **3 a** $x = -\frac{1}{3k+1}, y = -\frac{5k+2}{3k+1}, k \neq \frac{1}{3}$ **b** $x = \frac{3(3-k)}{3-k^2}, y = \frac{3(k-1)}{3-k^2}, k \neq \pm\sqrt{3}$

3 c $x = \frac{12}{11k-5}, y = \frac{-13k+7}{11k-5}, k \neq \frac{5}{11}$ **d** $x = \frac{31-33k}{3k^2+2k-5}, y = \frac{5k-6}{3k^2+2k-5}, k \neq -\frac{5}{3}, 1$ **4 a** Unique solution **b** Unique solution

4 c No unique solution **d** No unique solution **5 a** $c = 1, 0$. Lines are parallel. **b** $c = -2$. Lines are parallel.

6 a Consistent. Lines intersect giving unique solution. **b** Consistent. Same line giving infinite solutions. **c** Consistent. Same line giving infinite solutions.

7 $p = 3$ **8** $\lambda = -6, x = 1, y = -1$

Chapter 11 Exercise 4

1 a $x = -1, y = 9, z = -13$ **b** $x = 10, y = 10, z = -36$ **c** $x = 2, y = -4, z = -3$ **d** $x = 2, y = -1, z = 2$ **2 a** $x = -3, y = 2, z = 4$

2 b $x = -2, y = 1, z = 4$ **c** $x = -1, y = 1, z = 2$ **d** $x = 4, y = -5, z = 2$ **3 a** $x = 4, y = \frac{1}{3}, z = -\frac{2}{3}$ **b** $x = \frac{55}{47}, y = \frac{40}{47}, z = -\frac{64}{47}$

3 c $x = \frac{1}{2}, y = 1, z = -2$ **d** $x = 2, y = 3, z = -2$ **4 a** Determinant = 6. Unique solution. **b** Determinant = 0. No unique solution.

4 c Determinant = 0. No unique solution. **d** Determinant = 0. No unique solution. **5 a** $x = -1, y = -1, z = 1$ **b** $x = -\frac{3}{19}, y = -\frac{59}{19}, z = \frac{28}{19}$

5 c $x = -1, y = -2, z = 4$ **d** $x = \frac{1}{4}, y = \frac{1}{2}, z = 2$ **6 a** $x = \frac{74}{19}, y = -\frac{3}{19}, z = -\frac{9}{19}$ **b** $x = 2, y = -1, z = -1$

6 c $x = \frac{35}{66}, y = -\frac{43}{66}, z = -\frac{13}{33}$ **d** $x = 0, y = 2, z = 3$ **7 a** $x = \frac{5-24\lambda}{2}, y = \lambda, z = \frac{2\lambda-3}{4}$ **b** $x = -\frac{2}{15}, y = \frac{4}{15}, z = \frac{4}{3}$

7 c $x = \frac{19-5\lambda}{13}, y = \lambda, z = \frac{7\lambda-11}{13}$ **d** $x = \lambda, y = \mu, z = \frac{4-2\lambda-\mu}{3}$ **e** $x = \frac{7}{5}, y = 0, z = \frac{2}{5}$ **f** No solution. **g** No solution.

7 h $x = 4, y = 4, z = 6$ **8 a** $x = 2\lambda - 3, y = \lambda, z = \frac{5\lambda-10}{2}$ **b** $x = -\frac{5}{44}, y = \frac{15}{44}, z = -\frac{7}{22}$ **c** No solution.

8 d No solution. **e** $x = \frac{44-4\lambda}{11}, y = \lambda, z = \frac{3\lambda}{11}$ **f** $x = 1, y = 2, z = -3$ **9 a** $\begin{pmatrix} \frac{1}{5} & \frac{3}{5} & -\frac{2}{15} \\ -\frac{1}{10} & \frac{1}{5} & -\frac{1}{10} \\ -\frac{1}{6} & 0 & \frac{1}{18} \end{pmatrix}$ **b** $x = \frac{7}{15}, y = -\frac{2}{5}, z = -\frac{1}{9}$

10 a 0 **b** $c = 3$ **c** $x = \lambda, y = \lambda, z = -2\lambda$

12 $a = \frac{2-4b}{b+20}$ **13 a** $-2k^2 - 11k + 37$ **b** $k = -7.86, 2.36$ **14** $a = 36$

Chapter 11 Review Exercise

1 R is an $n \times p$ matrix R is an $m \times p$ matrix **2** $\lambda = 1$ or 6 **3 a** $\frac{1}{k^2+1} \begin{pmatrix} k & 1 \\ -1 & k \end{pmatrix}$ **b** $x = 1, y = -k$ **4** $x = 1, y = 8$

5 a $\begin{pmatrix} 1 & k+3 & 5 \\ 1 & 3 & k+1 \\ 1 & 1 & k \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ k+2 \\ 2k-1 \end{pmatrix}$ **b** $k = \frac{8}{3}$ **c** $z = \frac{k^2-5k-4}{8-3k}$ **d** If $k = \frac{8}{3}$ there is no solution.

5 d Otherwise $x = \frac{-k^3-2k^2+29k-22}{8-3k}, y = \frac{k^2-6k+14}{8-3k}, z = \frac{k^2-5k-4}{8-3k}$ **6** $a = -1, b = 3$ **7 a** $3p - 3q - r = 0$

7 b Solution is not unique. $x = \frac{17+3\lambda}{51}, y = \lambda, z = -\frac{5\lambda+11}{17}$ **8 b** $k = -3$ **9** $p = 3, q = -5$ **10 a** $c = -2.5, 0.5, 2$

10 b $\begin{pmatrix} 36 & 39 & 50 \\ 14 & 16 & 20 \\ 15 & 18 & 22 \end{pmatrix}$ **c** Since M is singular, A must be singular. **11** $k = 5$ **12 b** $c = -3$ **c** $x = -\frac{1+7\lambda}{2}, y = \lambda, z = \frac{11\lambda+7}{2}$

13 a $a = 7, b = 2$ **b** $x = -1, y = 2, z = -1$ **15** $a = 1$ **16** $a = 4, b = -1$ $y_1 = 16z_1 + 36z_2 - 58z_3$

17 $y_2 = -18z_1 + 11z_2 - 3z_3$ $y_3 = 13z_1 + 12z_2 - 17z_3$

Chapter 12 Exercise 1

1 a $a = 1$ $b = \frac{1}{2}$ $c = 6$ **b** $a = \frac{1}{3}$ $b = 0, \frac{1}{2}$ $c = -8$ **c** $a = \frac{4}{7}$ $b = -3$ **2 a** $PQ = i - 4j$ **b** $|PQ| = \sqrt{17}$

3 a $AB = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ **b** $|AB| = \sqrt{13}$ **4** $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ **5 a** $\sqrt{34}$ **b** $\sqrt{53}$ **c** $\sqrt{90}$ **d** $\sqrt{29}$ **e** $\sqrt{21}$ **f** $\sqrt{57}$ **6 a** Parallel **b** Parallel

6 c Not parallel **d** Not parallel **7 a** $c = 6$ **b** $c = -7$ **c** $c = 6$ **8** $\begin{pmatrix} \frac{13}{2} \\ \frac{13\sqrt{3}}{2} \end{pmatrix}$ **9** $\begin{pmatrix} \frac{-5}{\sqrt{62}} \\ \frac{-6}{\sqrt{62}} \\ \frac{1}{\sqrt{62}} \end{pmatrix}$ **10 a** Not parallel **b** Not parallel **c** Parallel

11 $PQ = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $QR = \begin{pmatrix} -4 \\ -6 \end{pmatrix}$ $PR = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$ $|PQ| = \sqrt{13}$ $|QR| = \sqrt{21}$ $|QR| = \sqrt{52}$ $|PR| = \sqrt{13}$ 12 **a** $R = \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}$

12 **b** $PQ = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$ $QR = \begin{pmatrix} -1 \\ 4 \\ 2 \end{pmatrix}$ $SR = \begin{pmatrix} 4 \\ -2 \\ -1 \end{pmatrix}$ $RP = \begin{pmatrix} -3 \\ -2 \\ -1 \end{pmatrix}$ **c** $|PQ| = \sqrt{21}$ $|QR| = \sqrt{21}$ $|SR| = \sqrt{21}$ $|RP| = \sqrt{14}$

12 **f** $PQ = \begin{pmatrix} \frac{4}{\sqrt{21}} \\ -\frac{2}{\sqrt{21}} \\ \frac{-1}{\sqrt{21}} \end{pmatrix}$ $QR = \begin{pmatrix} \frac{-1}{\sqrt{21}} \\ \frac{4}{\sqrt{21}} \\ \frac{2}{\sqrt{21}} \end{pmatrix}$ $SR = \begin{pmatrix} \frac{4}{\sqrt{21}} \\ -\frac{2}{\sqrt{21}} \\ \frac{-1}{\sqrt{21}} \end{pmatrix}$ $RP = \begin{pmatrix} \frac{-3}{\sqrt{14}} \\ \frac{-2}{\sqrt{14}} \\ \frac{-1}{\sqrt{14}} \end{pmatrix}$

14 $SU = -a - b$ 15 $a = -1, b = 1$ 16 $p = -\frac{4}{7}, q = \frac{4}{7}, r = -\frac{6}{7}$ 17 $x = -\frac{69}{53}, y = \frac{75}{53}$

Chapter 12 Exercise 2

1 **a** $3i + 4j - k$ **b** $6i - 2j - 9k$ **c** $-2i + 11j + 2k$ **d** $17i - 21j - 16k$ **e** $-3i - 6j + 49k$ **f** $20i - 25j - 84k$

1 **g** $-5mi + 27mj + 17mk$ 2 **a** $17i + 16j$ **b** $\sqrt{421}$ **c** 43.0° 3 $q = 1$ Ratio is 1:2 4 **a** $AB = b - a$ **b** $AC = \frac{1}{2}(b - a)$

4 **c** $CB = \frac{1}{2}(b - a)$ **d** $OC = \frac{1}{2}(a + b)$ 5 **a** $\begin{pmatrix} -5 \\ 20 \end{pmatrix}$ **b** $\sqrt{425}$ **c** $\begin{pmatrix} -\frac{5}{2} \\ 10 \end{pmatrix}$ **d** $\begin{pmatrix} -2 \\ 8 \end{pmatrix}$ 6 **a** $\begin{pmatrix} 6 \\ 10 \end{pmatrix}$ **b** $\sqrt{136}$ **c** $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$ **d** $\begin{pmatrix} \frac{3}{4} \\ \frac{5}{4} \end{pmatrix}$

7 **a** $\frac{1}{1+k}(b - a)$ **b** $\frac{k}{1+k}(a - b)$ **c** $(a - b)$ **d** $\frac{ka + b}{1 + k}$

8 **a i** $CD = -a$ **ii** $CA = -b - a$ **iii** $BD = b - a$ **iv** $AX = \frac{1}{3}b$ **v** $XD = \frac{2}{3}b$ **b** $AC = \begin{pmatrix} 6k \\ 4c \end{pmatrix}$ **c** $BX = \begin{pmatrix} -2k \\ 3c \end{pmatrix}$

9 **a** $BE = \frac{mb}{m + n + 3}$ 7 **b** $EF = \frac{nb}{m + n + 3}$ **c** $CF = \frac{-3b}{m + n + 3}$ **d** $AF = a + \frac{(m + n)b}{m + n + 3}$ **e** $ED = -a + \frac{(2m + 3n + 9)b}{m + n + 3}$

10 **a** $BC = b$ **b** $FH = b - a$ **c** $AH = b + c$ **d** $AG = a + b + c$ **e** $BH = b + c - a$ 12 **a** $DG = d - a - b$

12 **c** $FA = -b - c - d$ 14 **a** $AB = c, BC = -a, AC = c - a, OB = a + c$ **b** They are perpendicular.

Chapter 12 Exercise 3

1 **a** 11 **b** -1 **c** 29 **d** 2 **e** 7 **f** 54 **g** 40 **h** 25 2 **a** -5 **b** 6 **c** 30 **d** -9 **e** 3 **f** 55 **g** -26 **h** 1 3 **a** 58.7° **b** 86.6°

3 **c** 24.8° **d** 129° **e** 54.0° **f** 50.0° 4 **p** $\mathbf{q} = 14, \cos \theta = \sqrt{\frac{7}{19}}$ 5 **a** and **d**, **a** and **f**, **b** and **c**, **b** and **e**. 6 **a** $-\frac{3}{2}$ **b** 11 **c** $\frac{15}{2}$ **d** -3, 2

8 $-\frac{5}{\sqrt{35}}i + \frac{3}{\sqrt{35}}j + \frac{1}{\sqrt{35}}k$ 9 $x = 17.9$ or $x = 6.5$ 10 70.5° 16 It is a rectangle since \hat{ABC} is 90° but we do not know if $AB = BC$

Chapter 12 Exercise 4

1 **a** $-14i - 5j - 8k$ **b** $-14i - 5j - 8k$ **c** $14i + 5j + 8k$ **d** $28i + 10j + 16k$ **e** $14i + 5j + 8k$ **f** $42i + 15j + 24k$ **g** 24

2 **a** $\frac{21\sqrt{3}}{2}$ **b** $\frac{9\sqrt{39}}{2}$ **c** 6 7 **a** $-\frac{18}{\sqrt{817}}i + \frac{3}{\sqrt{817}}j + \frac{22}{\sqrt{817}}k$ **b** $\sqrt{\frac{817}{986}}$ 8 **a** $-\frac{2}{\sqrt{6}}i - \frac{1}{\sqrt{6}}j + \frac{1}{\sqrt{6}}k$ **b** $\sqrt{\frac{54}{55}}$

9 $\frac{-3}{\sqrt{19}}i + \frac{3}{\sqrt{19}}j - \frac{1}{\sqrt{19}}k$ 10 $\frac{\sqrt{74}}{2}$ units² 11 $\frac{\sqrt{341}}{2}$ units² 12 $\sqrt{6}$ units² 13 $\sqrt{756}$ units² 14 $\sqrt{850}$ units² 15 $\sqrt{234}$ units²

16 $PQ = 25i - 5j + 10k$ and $PS = -6i - 14j + 2k$ Area = $10\sqrt{1734}$ units²

Chapter 12 Review Exercise

1 **ii** $\sqrt{378}$ units² **iii** $-10.8i + 9.6j - 1.2k$ **iv** 29.6° 2 **a** $6i - 12j + (2p + 1)k$ **b** $p = 4$ 4 $\sqrt{2 - 2\cos \theta}$ 5 $\begin{pmatrix} 5\cos \theta + 3 \\ 5\sin \theta + 2 \end{pmatrix}$

6 0 8 0.70210 $\alpha = \frac{\pi}{2} - 2\theta$ 11 **a** A has coordinates (2, 4, 6) B has coordinates (6, -3, 0) C has coordinates (4, -7, -6)

11 **b** 50.2 units² 11 **c** $\begin{pmatrix} 3 \\ -\frac{3}{2} \\ 0 \end{pmatrix}$ **d** 96.3° **e** 22.6 units² 12 $m = \frac{10}{3}$

Chapter 13 Exercise 1

1 **a** $r = \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ **b** $r = i - 2j + \lambda(i - 4j - 2k)$ **c** $r = \begin{pmatrix} 4 \\ 4 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ -5 \\ 12 \end{pmatrix}$ **d** $r = 5i + 2j + k + \lambda(3i + 6j - k)$ **e** $r = -3i - j + \lambda(2i - j)$

1 **f** $r = \begin{pmatrix} -5 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -7 \end{pmatrix}$ 2 **a** $r = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -3 \\ -1 \end{pmatrix}$ **b** $r = \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 7 \\ -2 \\ 2 \end{pmatrix}$ **c** $r = \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 9 \\ -6 \end{pmatrix}$ **d** $r = \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -9 \\ 1 \end{pmatrix}$

2 **e** $r = \begin{pmatrix} 4 \\ -3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 0 \end{pmatrix}$ 3 **a** $r = \begin{pmatrix} -3 \\ -2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -7 \\ 3 \end{pmatrix}$ $x = 1 + 3\lambda, y = -2 + \lambda, z = -4 - 5\lambda$ $\frac{x-1}{3} = y + 2 = \frac{z+4}{-5}$

3 **b** $r = i - 2j - 4k + \lambda(3i + j - 5k)$ $x = -3 + 4\lambda, y = -2 - 7\lambda, z = 3 + 3\lambda$ $\frac{x+3}{4} = \frac{y+2}{-7} = \frac{z-3}{3}$

3 **c** $r = j + k + \lambda(i - 3k)$ $x = \lambda, y = 1, z = 1 - 3\lambda$ $y = 1, x = \frac{1-z}{3}$

3 **d** $r = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$ $x = 4 - \lambda, y = 1 + 2\lambda, z = 2\lambda$ $4 - x = \frac{y-1}{2} = \frac{z}{2}$

4 **a** $x = 1 - 2\lambda, y = -1 + 3\lambda, z = 2 - 2\lambda$ $\frac{1-x}{2} = \frac{y+1}{3} = \frac{2-z}{2}$ **b** $x = 2 + 3\mu, y = -5 - \mu, z = -1 + 4\mu$ $\frac{x-2}{3} = \frac{y+5}{-1} = \frac{z+1}{4}$

4 **c** $x = 2 + 4m, y = 8 - 7m, z = -1 + 6m$ $\frac{x-2}{4} = \frac{8-y}{7} = \frac{z+1}{6}$ **d** $x = 1 + 2n, y = -1 - 3n, z = 7 - n$ $\frac{x-1}{2} = \frac{y+1}{-3} = 7 - z$

4 **e** $x = 4 + 3s, y = 6 - 5s$ $\frac{x-4}{3} = \frac{6-y}{5}$ **f** $x = 1 + 2t, y = -6 - 5t$ $\frac{x-1}{2} = \frac{y+6}{-5}$

5 **a** $r = \begin{pmatrix} -7 \\ 6 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$ **b** $r = \begin{pmatrix} -4 \\ -5 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix}$ **c** $r = \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix} + m \begin{pmatrix} 5 \\ 3 \\ 4 \end{pmatrix}$ **d** $r = \begin{pmatrix} -4 \\ -1 \\ 5 \end{pmatrix} + n \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ 6 **a** $r = \begin{pmatrix} 3 \\ -5 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 3 \\ -3 \end{pmatrix}$

6 **b** $r = \begin{pmatrix} \frac{5}{2} \\ -1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -\frac{4}{3} \\ 2 \end{pmatrix}$ **c** $r = \begin{pmatrix} \frac{2}{5} \\ -\frac{5}{3} \\ -\frac{7}{2} \end{pmatrix} + \lambda \begin{pmatrix} \frac{4}{5} \\ -2 \\ \frac{3}{2} \end{pmatrix}$ **d** $r = \begin{pmatrix} -\frac{1}{6} \\ \frac{4}{3} \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ -9 \end{pmatrix}$ **e** $r = \begin{pmatrix} \frac{5}{3} \\ -\frac{2}{3} \\ \frac{1}{3} \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ **f** $r = \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 49 \\ -4 \\ 0 \end{pmatrix}$

7 **a** No **b** Yes **c** No **d** Yes **e** No **f** Yes 8 $r = \begin{pmatrix} 3 \\ 7 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ -9 \\ 2 \end{pmatrix}$ Position vector is $\begin{pmatrix} 6 \\ \frac{5}{2} \\ 0 \end{pmatrix}$

9 Crosses the xy plane at (-9, -7, 0) Crosses the yz plane at $(0, 11, -\frac{9}{2})$ Crosses the xz plane at $(-\frac{11}{2}, 0, -\frac{7}{4})$

10 **a** $r = 2i - j + 5k + \lambda(3i - j + k)$ Crosses the xy plane at (-13, 4, 0) Crosses the yz plane at $(0, -\frac{1}{3}, \frac{13}{3})$ Crosses the xz plane at (-1, 0, 4)

10 **b** $r = \begin{pmatrix} 2 \\ 6 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$ Crosses the xy plane at (9, -1, 0) Crosses the yz plane at (0, 8, 9) Crosses the xz plane at (8, 0, 1)

11 Crosses the xy plane at $(-\frac{29}{15}, \frac{14}{5}, 0)$ Crosses the yz plane at $(0, \frac{17}{4}, -\frac{29}{12})$ Crosses the xz plane at $(-\frac{17}{3}, 0, \frac{14}{3})$

Chapter 13 Exercise 2

1 **a** Skew **b** Intersect at the point (2, 1, 6) **c** Parallel **d** Skew **e** Skew **f** Parallel **g** Skew

2 **a** $\frac{\sqrt{26}}{2}$ **b** $\frac{\sqrt{393}}{3}$ **c** 1 **d** $\frac{\sqrt{154}}{2}$ **e** Lies on line. **f** $\frac{2\sqrt{35}}{7}$

3 $AB: r = \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ 7 \end{pmatrix}$ $\frac{x}{3} = \frac{y-1}{4} = \frac{z+2}{7}$ $AD: r = \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ $x = \frac{y-1}{2} = -z - 2$ Coordinates of C are (4, 7, 4)

4 **a** 60.5° **b** 36.3° **c** 71.2° **d** 88.4° **e** 62.8° 5 $a = -2$ Point of intersection is (1, 5, -3) 6 $r = \begin{pmatrix} -3 \\ 8 \end{pmatrix} + t \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ 7 $p = 34$

Chapter 13 Exercise 3

- 1 a** $r.\left(\frac{\frac{1}{\sqrt{18}}}{\frac{4}{\sqrt{18}}}-\frac{\frac{-1}{\sqrt{18}}}{\frac{1}{\sqrt{18}}}\right)=\frac{3}{\sqrt{18}}$ **b** $r.\left(\frac{1}{\sqrt{35}}i-\frac{3}{\sqrt{35}}j+\frac{5}{\sqrt{35}}k\right)=\frac{-8}{\sqrt{35}}$ **c** $r.\left(\frac{\frac{2}{\sqrt{5}}}{0}=\frac{6}{\sqrt{5}}$ **d** $r.\left(\frac{-1}{\sqrt{18}}i-\frac{4}{\sqrt{18}}j+\frac{1}{\sqrt{18}}k\right)=\frac{-15}{\sqrt{18}}$
- 1 e** $r.\left(\frac{1}{\sqrt{74}}i-\frac{8}{\sqrt{74}}j+\frac{3}{\sqrt{74}}k\right)=\frac{-19}{\sqrt{74}}$ **f** $r.\left(\frac{-3}{\sqrt{19}}i+\frac{3}{\sqrt{19}}j+\frac{1}{\sqrt{19}}k\right)=\frac{3}{\sqrt{19}}$ **g** $r.\left(\frac{10}{\sqrt{1133}}i-\frac{3}{\sqrt{1133}}j+\frac{32}{\sqrt{1133}}k\right)=\frac{-63}{\sqrt{1133}}$
- 1 h** $r.\left(\frac{\frac{7}{\sqrt{354}}}{\frac{17}{\sqrt{354}}}-\frac{\frac{4}{\sqrt{354}}}{\frac{11}{\sqrt{354}}}\right)=\frac{32}{\sqrt{354}}$ **i** $r.\left(\frac{\frac{20}{\sqrt{557}}}{\frac{6}{\sqrt{557}}}-\frac{\frac{11}{\sqrt{557}}}{\frac{1}{\sqrt{557}}}\right)=\frac{86}{\sqrt{557}}$ **j** $r.\left(\frac{-3}{\sqrt{26}}i-\frac{4}{\sqrt{26}}j+\frac{1}{\sqrt{26}}k\right)=0$ **k** $r.\left(\frac{3}{\sqrt{13}}i-\frac{2}{\sqrt{13}}j\right)=\frac{15}{\sqrt{13}}$
- 1 l** $r.\left(\frac{-7}{\sqrt{65}}i+\frac{4}{\sqrt{65}}k\right)=0$ **2 a** $r.(5j-2k)=-2$ **b** $r.(5i-4j+15k)=42$ **c** $r.\left(\frac{-15}{3}\right)=-3$ **d** $r.(i+11j-9k)=16$
- 3 a** $x-2y+7z=9$ **b** $4x-y=-6$ **c** $15x+13y-8z=-38$ **d** $-4x+3y+8z=21$ **4** $r.\left(\frac{-1}{-3}\right)=10$
- 5 a** $\frac{1}{\sqrt{30}}i+\frac{2}{\sqrt{30}}j-\frac{5}{\sqrt{30}}k$ **b** $\frac{4}{\sqrt{42}}i-\frac{1}{\sqrt{42}}j+\frac{5}{\sqrt{42}}k$ **c** $\frac{-3}{7}i-\frac{6}{7}j+\frac{2}{7}k$ **d** $\frac{5}{\sqrt{45}}i-\frac{2}{\sqrt{45}}j+\frac{4}{\sqrt{45}}k$ **7** $r.(13i-4j-11k)=31$
- 8** The direction normals are equal. Distance $\frac{8}{\sqrt{19}}$ units.
- 9 b** $r.(i+5j+k)=1$ **c** Distance of π_1 to origin is $\frac{8}{\sqrt{27}}$ units Distance of π_2 to origin is $\frac{1}{\sqrt{27}}$ units Distance between π_1 and π_2 is $\frac{7}{\sqrt{27}}$ units
- 10** Distance of P_1 to origin is $\frac{41}{\sqrt{65}}$ units Distance of P_2 to origin is $\frac{14}{\sqrt{65}}$ units Distance between P_1 and P_2 is $\frac{55}{\sqrt{65}}$ units
- 11** $r.(i-k)=1$ r_1 is not contained in the plane. r_2 is contained in the plane.
- 12** $r.\left(\frac{8}{\sqrt{89}}i-\frac{5}{\sqrt{89}}k\right)=\frac{-17}{\sqrt{89}}$ Distance of plane from the origin is $\frac{17}{\sqrt{89}}$ units.
- 13** $r.\left(\frac{2}{-3}\right)=\frac{-13}{\sqrt{17}}$ Distance of plane from the origin is $\frac{13}{\sqrt{17}}$ units. **14** $r=(4i+3j+7k)+\lambda(2i+2j-5k)$
- 15 b** $r.(i+j-k)=-9$ **c** Distance of P_1 to origin is $\frac{8}{\sqrt{3}}$ units Distance of P_2 to origin is $\frac{9}{\sqrt{3}}$ units Distance between P_1 and P_2 is $\frac{17}{\sqrt{3}}$ units
- 16 a** The line and the plane intersect. **b** The line and the plane are parallel. **c** The line is contained in the plane. **d** The line and the plane intersect.

Chapter 13 Exercise 4

- 1 a** $\left(9,\frac{1}{2},-\frac{2}{3}\right)$ **b** $\left(4,\frac{1}{2},-2\right)$ **c** $(3,-2,-9)$ **d** $\left(\frac{51}{7},\frac{1}{7},1\right)$ **e** $\left(\frac{20}{3},\frac{-13}{9},\frac{4}{3}\right)$ **f** $(16,-10,-7)$
- 2 a** 45.0° **b** 23.2° **c** 11.0° **d** 64.5° **e** 90° **f** 55.9° **3 a** 11.7° **b** 57.8° **c** 17.6° **d** 32.5° **e** 34.1° **f** 5.51°
- 4 a** $r=(4i+j)+\lambda(14i+17j+13k)$ **b** $r=\left(\frac{2}{-2}\right)+\lambda\left(\frac{7}{-1}\right)$ **c** $r=(7i+3j)+\lambda(2i-k)$ **d** $r=(i-6j)+\lambda(-3i-2j+k)$
- 4 e** $r=(8i+14j)+\lambda(-37i+61j+10k)$ **f** $r=\left(\frac{56}{25}i+\frac{29}{25}j\right)+\lambda(36i-26j-25k)$ **6** 48.5° **7 a** $r=\left(\frac{1}{2}\right)+\lambda\left(\frac{3}{-3}\right)$ **b** $(-2,1,0)$
- 8 a** $a=-3$ **b** 74.2° **c** 13.9 units.

Chapter 13 Review Exercise

- 1 i** $\frac{3}{\sqrt{178}}i-\frac{12}{\sqrt{178}}j+\frac{5}{\sqrt{178}}k$ **ii** $r.\left(\frac{3}{\sqrt{178}}i-\frac{12}{\sqrt{178}}j+\frac{5}{\sqrt{178}}k\right)=\frac{7}{\sqrt{178}}$ **iii** 73.5° **2 ii** $\frac{\sqrt{30}}{2}$ units² **iii** $\frac{1}{\sqrt{30}}i-\frac{2}{\sqrt{30}}j-\frac{5}{\sqrt{30}}k$

- 2 iv** AD has equation $x-1=\frac{1-y}{3}=\frac{1-z}{4}$ BD has equation $2x-4=\frac{4+2y}{-9}=\frac{6+2z}{-7}$ **v** 16.4° **3 b** $3x-2y+z=5$
- 3 c** $r=\begin{pmatrix}2\\1\\1\end{pmatrix}+\lambda\begin{pmatrix}-1\\-2\\-1\end{pmatrix}$ **4 ii** $\left(\frac{2}{21},\frac{-4}{21},\frac{-8}{21}\right)$ **iii** $r.\begin{pmatrix}4\\-8\\5\end{pmatrix}=0$ **iv** $\left(-33,\frac{-51}{2},4\right)$ **5 i** $\left(1,-\frac{13}{5},\frac{12}{5}\right)$ **ii** $(6,-1,4)$ **iii** $r=\begin{pmatrix}6\\-1\\4\end{pmatrix}+\lambda\begin{pmatrix}25\\8\\6\end{pmatrix}$
- 5 iv** 116° **6 a** $\frac{x-2}{1}=\frac{y-5}{1}=\frac{z+1}{1}$ **b** $\left(\frac{1}{3},\frac{10}{3},\frac{-8}{3}\right)$ **c** $\left(\frac{-4}{3},\frac{5}{3},\frac{-13}{3}\right)$ **d** 8.52 **7** $r=\begin{pmatrix}-1\\-3\\0\end{pmatrix}+\lambda\begin{pmatrix}-1\\1\\1\end{pmatrix}$ **8 a** $AB=\begin{pmatrix}-1\\-3\\1\end{pmatrix}BC=\begin{pmatrix}1\\0\\1\end{pmatrix}$
- 8 b** $-i+j+2k$ **c** $\frac{\sqrt{6}}{2}$ **d** $r.\begin{pmatrix}-1\\1\\2\end{pmatrix}=3$ **e** $x=2-\lambda$ $y=-1+\lambda$ $z=-6+2\lambda$ **f** $3\sqrt{6}$ **g** $\frac{-i}{\sqrt{6}}+\frac{j}{\sqrt{6}}+\frac{2k}{\sqrt{6}}$ **h** $(-4,5,6)$
- 9 i** A lies in the plane. B does not lie in the plane **ii** $\frac{2-x}{2}=\frac{y+3}{4}=z-8$ **iii** 43.6° **iv** 3.16 units. **v** $r.(i+3j-10k)=-87$
- 10 a** $-2i+2j-k$ **b i** $n_1=6i+3j-2k$ and $n_2=-2i+2j-k$ **ii** 79.0° **d i** $(8,-20,-12)$

- 11 i** $r=(2i+j+4k)+\lambda(3i+j)$ **ii** $(-4,-1,4)$ **iii** 0.716 **iv** $(0,-13,18)$ **v** $\left(-\frac{13}{3},0,\frac{28}{3}\right)$ **vi** $r=\begin{pmatrix}0\\-13\\18\end{pmatrix}+\lambda\begin{pmatrix}-\frac{13}{3}\\13\\-\frac{26}{3}\end{pmatrix}$

- 12 a ii** $r=(-j-k)+\lambda(3i+11j+k)$ **b** $c=-2$. Line of intersection. **c ii** $\frac{7\sqrt{2}}{6}$ **13** $r=\begin{pmatrix}7\\0\\-4\end{pmatrix}+\lambda\begin{pmatrix}7\\1\\5\end{pmatrix}$

- 14 a** $r=(2i+3j+7k)+t(3i+j+3k)$ **b** $(8,5,13)$ **c** $2x+3y-4z+4=0$ **e i** $i+j-k$ **ii** $PO=i-2j-4k$ **iii** $\frac{12}{\sqrt{14}}$
- 15 b** $\frac{4}{\sqrt{21}}$ **c** $r.(i+4j+2k)=-9$ **d** $\frac{13}{\sqrt{21}}$ **16 a** $\left(4,\frac{5}{2},\frac{13}{2}\right)$ **b** $x+2y=9$ **c** $\left(\frac{37}{5},\frac{4}{5},\frac{9}{5}\right)$ **d** $(6.63,3.50,4.65)$

Chapter 14 Exercise 1

- 1** $5x$ **2** $10x$ **3** $-2x$ **4** $2x^2$ **5** $6x^2$ **6** x^3 **7** x^4 **8** x^5 **9** $3x^3$ **10** $4x^{-1}$

Chapter 14 Exercise 2

- 1** x^2-x+c **2** $\frac{1}{3}x^3+c$ **3** $\frac{1}{4}x^4+c$ **4** $\frac{1}{5}x^5+c$ **5** $2x^3-5x+c$ **6** $2x^4+2x^2-3x+c$ **7** $\frac{5}{3}x^3-4x+c$ **8** $-x^{-1}+c$
- 9** $\frac{2}{3}x^{\frac{3}{2}}+c$ **10** $3x^{\frac{1}{2}}+c$ **11** $7x+2x^{-2}+c$ **12** $2x^{\frac{1}{2}}+c$ **13** $\frac{2}{7}x^7-x^5+c$ **14** $2x^{\frac{5}{2}}+2x^{-2}+c$ **15** x^4+2x^2-9x+c
- 16** $x-x^2+2x^3-\frac{1}{4}x^4+c$ **17** $-3x^{-2}+c$ **18** $y=-\frac{1}{2}x^{-4}+c$ **19** $y=\frac{2}{3}x^{\frac{3}{2}}-2x^{\frac{1}{2}}+c$ **20** $y=8x-\frac{9}{2}x^{\frac{3}{2}}+c$ **21** $y=4x^4-12x^2+c$
- 22** $y=\frac{2}{3}x^3-\frac{21}{2}x^2+27x+c$ **23** $y=3x^3-12x^2+16x+c$ **24** $y=-\frac{1}{2}x^{-2}+\frac{5}{4}x^{-4}+c$ **25** $y=\frac{8}{7}x^{\frac{7}{2}}-\frac{14}{3}x^{\frac{3}{2}}+c$ **26** $y=\frac{14}{19}x^{\frac{19}{2}}-2x^{\frac{3}{2}}+c$
- 27** $y=-6p^{-2}+c$ **28** $y=\frac{32}{9}k^{\frac{3}{2}}+c$ **29** $y=\frac{1}{6}z^6-\frac{1}{2}z^2+c$ **30** $y=-t^{-1}-3t^{-2}-3t^{-3}+c$ **31** $y=\frac{2}{3}t^{\frac{1}{2}}-\frac{4}{9}t^{\frac{3}{2}}+c$

Chapter 14 Exercise 3

- 1** $y=6x-4$ **2** $y=2x^2+3$ **3** $y=4x^2-3x-18$ **4** $y=-x^2+5x$ **5** $y=x^4-2x^3+7x+3$ **6** $y=\frac{4}{3}x^3-6x^{-1}-\frac{509}{6}$
- 7** $y=16x^{\frac{1}{2}}-46$ **8** $y=\frac{1}{7}t^7-\frac{3}{5}t^5-\frac{4}{3}t^3+\frac{818}{105}$ **9** $Q=\frac{2}{21}p^{\frac{2}{3}}-\frac{8}{33}p^{\frac{2}{3}}+2$

Chapter 14 Exercise 4

- 1** $\frac{1}{4}x^4-2\ln|x|+c$ **2** $4e^x-\cos x+c$ **3** $5\ln|x|-\sin x+c$ **4** $-6\cos x-\frac{6}{5}x^5+c$ **5** $8\cos x+7e^x+c$
- 6** $5e^x+2\cos x+3\ln|x|+c$ **7** $\frac{1}{3}e^x-\frac{5}{2}\ln|x|-7\cos x+c$ **8** $\frac{e^x}{15}-10x^{\frac{3}{2}}+\sin x+c$

Chapter 14 Exercise 5

- 1 $-\frac{1}{5}\cos 5x + c$ 2 $\frac{1}{6}\sin 6x + c$ 3 $-\frac{1}{2}\cos 2x + c$ 4 $-2\cos\frac{1}{2}x + c$ 5 $2\sin 4x + c$ 6 $2\cos 3x + c$ 7 $-\frac{5}{2}\sin 2x + c$ 8 $\frac{1}{6}e^{6x} + c$
- 9 $\frac{1}{5}e^{5x} + c$ 10 $e^{4x} + c$ 11 $\frac{4}{3}e^{6t} + c$ 12 $-\frac{5}{6}e^{6p} + c$ 13 $4x^2 - \frac{1}{2}e^{2x} + c$ 14 $-2e^{-2x} + c$ 15 $y = \frac{1}{2}\ln|2x - 3| + c$
- 16 $y = \frac{1}{8}\ln|8x + 7| + c$ 17 $y = 2\ln|2x - 5| + c$ 18 $y = \frac{1}{18}(3x - 1)^6 + c$ 19 $y = \frac{1}{28}(4x - 7)^7 + c$ 20 $y = -\frac{1}{8}(4x + 3)^{-2} + c$
- 21 $y = -\frac{1}{10}(3 - 2x)^5 + c$ 22 $y = -\frac{2}{3}(3x - 2)^{-2} + c$ 23 $y = -\frac{3}{2}(2t - 1)^{-1} + c$ 24 $y = \frac{4}{3}\ln|3x - 1| + c$ 25 $y = 2\ln|3x - 5| + c$
- 26 $y = -8\ln|4 - p| + c$ 27 $y = -3\ln|6 - t| + c$ 28 $\frac{3}{2}e^{4x} + c$ 29 $-\frac{1}{3}\cos 3x - 2x^2 + c$ 30 $-\frac{1}{2}e^{-8x} - 2\sin 2x + c$
- 31 $\frac{1}{2}\ln|2x - 1| + \frac{1}{18}(3x + 4)^6 + c$ 32 $2x^3 - \frac{2}{3}\ln|3x + 2| + c$

Chapter 14 Exercise 6

- 1 3 2 38 3 20 4 0 5 $\frac{2}{3}$ 6 $-\frac{8}{3}$ 7 201 8 216468 9 1490 10 $\frac{2}{3}$ 11 0 12 $\frac{1 - 3\pi}{3}$ 13 21 14 -200 15 312.6
- 16 0.490 17 0.0429 18 0.549 19 1.85 20 -3.47 21 8.56 22 $-\frac{1}{4}\cos p - 3p^2 + \frac{1}{4}$ 23 $2\ln|2k + 1|$

Chapter 14 Exercise 7

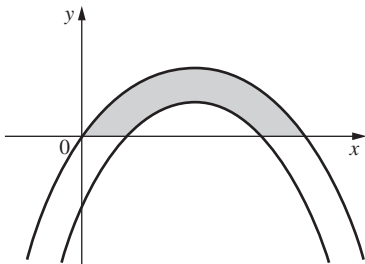
- 1 32 2 $\frac{343}{6}$ 3 50 4 $\frac{85}{4}$ 5 2 6 6.39 7 5.55 8 0.619 9 22 10 1.69 11 0.825 12 2.19 13 27 14 10.4 15 2 16 2.27
- 17 $\frac{4}{3}$ 18 9 19 $\frac{4}{3}$ 20 1.48 21 $\frac{2}{3}\ln\left|\frac{3p + 5}{8}\right|$ 22 $-\frac{1}{2}e^{-2p} + \frac{1}{3}p^3$ 23 $k = 4$ 24 $a = 1$

Chapter 14 Exercise 8

- 1 $\frac{64}{3}$ 2 61 3 16 4 $\frac{85}{4}$ 5 $\frac{253}{12}$ 6 $\frac{2401}{16}$ 7 $\frac{863}{6}$ 8 22.1 9 $\frac{71}{3}$ 10 408 11 $\frac{21}{4}$ 12 2 13 6 14 15.3

Chapter 14 Exercise 9

- 1 $\frac{1}{6}$ 2 $\frac{1}{8}$ 3 $\frac{1}{2}$ 4 $\frac{1}{3}$ 5 $\frac{125}{6}$ 6 $\frac{407}{4}$ 7 36 8 6.43 9 $\frac{160}{3}$
- 10 1.60 11 1.85 12 3.08 13 $\frac{5}{2}$ 14 3.92 15 $\frac{8}{3}$ 16 $\frac{32}{3}$
- 17 $\frac{256}{3}$ 18 18 19 7.45 20 3.62 21 4.53 22 3.21



Chapter 14 Review Exercise

- 1 **a** $\frac{4}{3}x^3 - 7x + c$ **b** $3x^3 + 2x^2 - 5x + c$ **c** $-4x^{-2} + c$ **d** $-\frac{1}{6}(3 - 2x)^3 + c$ 2 **a** $y = -\frac{1}{x} + \frac{3}{2}x^{-4} + c$ **b** $y = p^3 - \frac{1}{8}p^8 + c$
- 2 **c** $y = \frac{3}{8}t^2 - \frac{1}{2}t^{\frac{1}{2}} + c$ 3 $y = -\frac{3}{2}x^2 + 8x - 2$ 4 **a** $4e^x + \cos x + c$ **b** $7\sin x - 4\ln|x| + c$ **c** $\frac{1}{3}e^{6x} - 5\ln|x| - 4\cos x + c$
- 5 **a** $3\sin 2x + c$ **b** $2e^{2x} + c$ **c** $\frac{1}{2}\ln|4x - 3| + c$ **d** $\frac{1}{21}(3x - 2)^7 + c$ **e** $\frac{7}{3}e^{3x} + \frac{1}{3}(3x - 4)^{-4} + c$ 6 $\frac{4x^{\frac{5}{2}}}{5} + \frac{3}{x} + c$
- 7 **a** $\frac{382}{25}$ **b** $\frac{1}{2} + \frac{1}{4}\pi$ **c** $3\sin 2k$ 8 **a** $e^5 - e^2$ **b** 36 **c** $\frac{2}{3}$ 9 $\frac{3}{2}\ln(2p - 5)$
- 10 $\frac{407}{4}$ 11 **a** 0.753 **b** 2.45 **c** 1.78 12 **a** $\frac{9}{2}$ **b** 24.3 13 **a** 13.3 **b** 1.93 14 30.2

Chapter 15 Exercise 1

- 1 $-\cos\left(\theta - \frac{3\pi}{4}\right) + c$ 2 $\frac{1}{3}\sin\left(3x + \frac{\pi}{4}\right) + c$ 3 $\frac{1}{32}e^{32x-7} + c$ 4 $2e^{\sin 2} - 2$ 5 $\frac{1}{4}\ln|8x - 9| + c$ 6 $\frac{1}{54}(x^6 - 9)^9 + c$
- 7 $\frac{1}{16}[(2p^4 + 1)^2 - 1]$ 8 $\frac{1}{3}(1 + x^2)^{\frac{3}{2}} + c$ 9 $\frac{1}{12}(4x^2 - 3)^{\frac{3}{2}} + c$ 10 $\frac{1}{12}(3\tan x + 4)^4 + c$ 11 $(x^2 - 1)^{\frac{1}{2}} + c$ 12 $\frac{1}{10}[1 - (2\cos 0.5 - 1)^5]$
- 13 $\frac{1}{24}[(6e^{2a} - 7)^2 + 1]$ 14 $-\frac{1}{3}(\cos 2x - 1)^{\frac{3}{2}} + c$ 15 $\frac{1}{12}(x^2 + 2x - 4)^6 + c$ 16 $\ln|x^2 - 3x + 5| + c$ 17 $\frac{3}{2(\cos x + 8)^2} + c$

- 18 $\frac{1}{2}\ln|2e^x - 4| + c$ 19 $\frac{1}{3}\ln|3\sin x - 12| + c$ 20 $\frac{2}{15}(3x^3 + 6x - 19)^{\frac{3}{2}} + c$ 21 $\frac{1}{21}(1 - 3\cos 2x)^{\frac{3}{2}} + c$ 22 $\frac{1}{6}\ln|3\tan 2x - 7| + c$

- 23 $\frac{15}{128}$ 24 $\frac{1}{3}\ln|3x^2 - 3x + 4| + c$ 25 $-\frac{1}{9(3x^2 - 3x + 4)^3} + c$ 26 $(\ln|p|)^2$ 27 $\ln|e^{2x} + 1| + c$ 28 $\frac{1}{6}\ln|2p^2 + 2p - 5|$

Chapter 15 Exercise 2

- 1 $\frac{1}{3}\tan^{-1}\frac{x}{3} + c$ 2 $\sin^{-1}\frac{x}{5} + c$ 3 $\cos^{-1}\frac{x}{6} + c$ 4 $3\tan^{-1}\frac{x}{3} + c$ 5 $2\sin^{-1}\frac{x}{2\sqrt{2}} + c$ 6 $\frac{\sqrt{3}}{3}\tan^{-1}\frac{p\sqrt{3}}{3} - \frac{\pi\sqrt{3}}{18}$ 7 0.0203
- 8 $\frac{\sqrt{3}}{3}\sin^{-1}x\sqrt{3} + c$ 9 $\sin^{-1}(x - 1) + c$ 10 $\frac{1}{8}\tan^{-1}\left(\frac{x + 1}{2}\right) + c$ 11 $\frac{1}{3}\tan^{-1}\left(\frac{x + 3}{3}\right) + c$ 12 $5\cos^{-1}\left(\frac{x + 2}{3}\right) + c$ 13 $\frac{1}{6}\tan^{-1}\left(\frac{3x + 1}{2}\right) + c$
- 14 $\frac{2}{\sqrt{11}}\tan^{-1}\left(\frac{2x + 3}{\sqrt{7}}\right) + c$ 15 $4\sin^{-1}\left(\frac{2x - 3}{7}\right) + c$ 16 $\frac{1}{3}\sin^{-1}\left(\frac{x - 1}{2\sqrt{3}}\right) + c$ 17 0.0623 18 $\sin^{-1}\left(\frac{p + 3}{\sqrt{3}}\right) - \sin^{-1}\left(\frac{\sqrt{3}}{3}\right)$

Chapter 15 Exercise 3

- 1 $\sin x - \frac{1}{3}\sin^3 x + c$ 2 $-\frac{1}{2}\cos 2x + \frac{1}{6}\cos^3 2x + c$ 3 $-\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + c$ 4 $\frac{1}{2}\tan 2p - p$ 5 $\frac{x}{2} + \frac{\sin 4x}{8} + c$
- 6 $\frac{x}{2} - \frac{\sin 4x}{8} + c$ 7 $\frac{1}{32}(12x - 8\sin 2x + \sin 4x) + c$ 8 $-\cos x + \frac{4}{3}\cos^3 x - \frac{6}{5}\cos^5 x + \frac{4}{7}\cos^7 x - \frac{1}{9}\cos^9 x + c$ 9 $\frac{1}{6}\tan^3 3x + \frac{1}{3}\ln|\cos 3x| + c$
- 10 $\frac{1}{64}(8x - \sin 4x) + c$ 11 $\frac{1}{4}\tan^4 x + \frac{1}{6}\tan^6 x + c$ 12 $-\frac{1}{6}\cos^3 2x + \frac{1}{10}\cos^5 2x + c$ 13 $\frac{1}{3}\sin^3 p - \frac{1}{5}\sin^5 p$

Chapter 15 Exercise 4

- 1 $\frac{(x + 2)^5}{5} + c$ 2 $\frac{(2 + 7x)^4}{28} + c$ 3 $-\sqrt{1 - 2x} + c$ 4 $\frac{-3}{4(2x + 1)^2} + \frac{(1 + 2x)^{\frac{3}{2}}}{3} + c$ 5 $\frac{(3 + 5x)^{\frac{3}{2}}}{30} + c$
- 6 $\frac{-2}{3}(1 - x)^{\frac{3}{2}} - 2(1 - x)^{\frac{1}{2}} - \frac{1}{1 - x} + c$ 7 $\frac{3}{4}\sin\left(4x - \frac{\pi}{2}\right) + c$ 8 $\tan^{-1} 2x + c$ 9 $\frac{1}{4}\ln|3 - 4\cos x| + c$ 10 $-\frac{1}{2}\tan\left(\frac{\pi}{3} - 2x\right) + c$
- 11 $-\frac{2}{3}\cos(3x + \alpha) + c$ 12 $\frac{1}{4}e^{4x+1} + c$ 13 $\frac{2^x}{\ln 2} + c$ 14 $\frac{1}{3}\ln|3x + 1| + c$ 15 $\ln|x^2 + 4| + c$ 16 $\frac{1}{2}\ln|x^2 + 2x + 3| + c$
- 17 $\frac{1}{4}\ln|x^4 + 3| + c$ 18 $\sin^{-1} 3x + c$ 19 $\frac{3}{4}\tan^{-1} 2x + c$ 20 $\frac{1}{14}(x^2 + 6x - 8)^7 + c$ 21 $\frac{1}{10}(\sin 2x + 3)^5 + c$ 22 $2e^{1 - \cot^{\frac{1}{2}} x} + c$
- 23 $\frac{1}{12}(1 + x^{\frac{3}{2}})^8 + c$ 24 $-\frac{1}{4}\cos^4 x + c$ 25 $\frac{1}{2(\cot x - 3)^2} + c$ 26 $\ln|e^x + 2| + c$ 27 $2(e^x + 2)^{\frac{1}{2}} + c$ 28 $\frac{1}{32}(48x - 4\sin 4x + \sin 8x) + c$
- 29 $\frac{1}{2}\ln|x^2 + 2x + 3| + c$ 30 $\sqrt{2}\tan^{-1}\left(\frac{x + 1}{\sqrt{2}}\right) + c$ 31 $2\sin^{-1}\left(\frac{x - 2}{3}\right) + c$ 32 $2(-x^2 + 4x + 5)^{\frac{1}{2}} + c$ 33 $\frac{1}{5}(\sin^2 x + 3)^5 + c$

Chapter 15 Exercise 5

- 1 $\frac{(x^2 + 3)^6}{12} + c$ 2 $\frac{1}{4}\ln|6x^2 + 4x - 13| + c$ 3 $-\sqrt{1 - \sin 2x} + c$ 4 $\frac{2}{15}(x - 2)^{\frac{3}{2}}(3x + 4) + c$ 5 $\frac{(2p - 1)^{\frac{1}{2}}(p + 1) - 2}{3}$
- 6 $\frac{(2x + 1)^{\frac{1}{2}}(x + 8)}{3} + c$ 7 $\tan^{-1} x^2 + c$ 8 $\frac{2(3x - 4)^{\frac{3}{2}}(9x + 38)}{13} + c$ 9 $\frac{(p - 2)^4(8p - 1) - 7}{20}$ 10 $\frac{1 - 6x}{24(2x - 1)^3} + c$ 11 $\frac{1 - 5x}{10(x - 3)^5} + c$
- 12 $\frac{2(p - 2)^{\frac{3}{2}}}{3} + 4(p - 2)^{\frac{1}{2}} - \frac{26\sqrt{7}}{3}$ 13 $\frac{(x + 5)^2}{2} - 15(x + 5) + 75\ln|x + 5| + \frac{125}{x + 5} + c$ 14 $\frac{2(5p + 2)^{\frac{3}{2}}(15p - 4)}{375} - \frac{416\sqrt{3}}{125}$
- 15 $\frac{x^2 - 4x + 8}{x - 2} + c$ 16 $\frac{1}{2}\tan^{-1}(2\tan x) + c$ 17 $\frac{3}{4}[\sin^{-1} x + 2x\sqrt{1 - x^2}] + c$ 18 $\frac{1}{2}\tan^{-1}\left(2\tan\frac{x}{2}\right) + c$
- 19 $2\sin^{-1}\frac{p}{2} + p\cos\left(\sin^{-1}\frac{p}{2}\right) - 2\sin^{-1}\frac{1}{4} + \frac{1}{2}\cos\left(\sin^{-1}\frac{1}{4}\right)$ 20 $\frac{1}{\sqrt{5}}\tan^{-1}(\sqrt{5}\tan x) + c$ 21 $\frac{75}{32}\left[\tan^{-1}\frac{4}{25}\left(5\tan\frac{p}{2} + 3\right)\right] - \frac{75}{32}\tan^{-1}\frac{12}{25}$
- 22 $\frac{-16}{1 + \tan 2x} + c$ 23 $\frac{8}{3n}\tan^{-1}\sqrt{x^n - 1} + c$

Chapter 15 Exercise 6

- 1 $x\sin x + \cos x + c$ 2 $\frac{e^{2x}}{4}(2x - 1) + c$ 3 $\frac{x^5}{25}(5\ln x - 1) + c$ 4 $\frac{-x\cos 2x}{2} + \frac{\sin 2x}{4} + c$ 5 $\frac{(p + 1)^{10}}{110}(p - 10) - \frac{1}{11}$
- 6 $-x^2\cos x + 2x\sin x + 2\cos x + c$ 7 $\frac{e^{2x}}{4}(2x^2 - 2x + 1) + c$ 8 $\frac{x^3}{9}(3\ln 3x - 1) + c$ 9 $x^3\ln 8x - \frac{x^3}{3} + c$ 10 $\frac{-e^{-3x}}{27}(9x^2 + 6x + 2) + c$

11 $\frac{e^x}{2}(\cos x + \sin x) + c$ **12** $x \sin^{-1} x + (1 - x^2)^{\frac{1}{2}} + c$ **13** $x \tan^{-1} x - \frac{1}{2} \ln|1 + x^2| + c$ **14** $e^{2x}(x - 1) + c$ **15** $\frac{e^{3x}}{10}(\sin x + 3 \cos x) + c$

16 $\frac{e^{2x}}{13}(2 \sin 3x - 3 \cos 3x) + c$ **17** $\frac{e^x}{5}(\sin 2x - 2 \cos 2x) + c$ **18** $\frac{x^{n+1}}{(n+1)^2}[(n+1) \ln x - 1] + c$ **19** $\frac{e^{ax}}{a^2 + b^2}(a \sin bx - b \cos bx) + c$

20 $\frac{1}{4(n+1)^2}[(2p+1)^{n+1}(2np-1) - 2n+1]$

Chapter 15 **Exercise 7**

1 $-(1 - x^2)^{\frac{1}{2}} + \sin^{-1} x + c$ **2** $\frac{3}{2} \ln|x^2 + 4| + 2 \tan^{-1} \frac{x}{2} + c$ **3** $\frac{1}{2} \ln|x^2 + 3| + \frac{5\sqrt{3}}{3} \tan^{-1} \frac{x\sqrt{3}}{3} + c$ **4** $2 \ln|x^2 + 4x + 8| - \frac{1}{2} \tan^{-1} \left(\frac{x+2}{2} \right) + c$

5 $\ln|x^2 + 4x + 6| - \frac{\sqrt{2}}{2} \tan^{-1} \left(\frac{x+2}{\sqrt{2}} \right) + c$ **6** $2(-x^2 - 6x - 4)^{\frac{1}{2}} + \sin^{-1} \left(\frac{x+3}{\sqrt{5}} \right) + c$ **7** $x - 7 \ln|x + 4| + c$ **8** $\frac{x^2}{2} - 3x + 10 \ln|x + 3| + c$

Chapter 15 **Exercise 8**

1 $\frac{\sin 3x}{3} - \frac{\cos 2x}{2} + c$ **2** $\frac{1}{3}(8x^{\frac{3}{2}} + 8(x+1)^{\frac{3}{2}} + (1-3x)^4) + c$ **3** $\frac{1}{3} \ln|3x^2 + 1| + c$ **4** $2 \tan^{-1} 2x + c$ **5** $\frac{1}{2} \tan \left(2x - \frac{\pi}{3} \right) + c$

6 $2\sqrt{1 + \sin x} + c$ **7** $2e^{x^2+x+5} + c$ **8** $\frac{1}{(n+1) \cos^{-n-1} x} + c$ **9** $\tan^{-1} 2(x+1) + c$ **10** $2 \sin^{-1} \left(\frac{x+4}{5} \right) + c$ **11** $\frac{(1-x)^8}{36}(7-8x) + c$

12 $x + \frac{25}{x+5} - 10 \ln|x+5| - \frac{25}{x+5} + c$ **13** $\frac{4(3x-4)^{\frac{3}{2}}}{135}(9x+8) + c$ **14** $\frac{-(5p+1)}{10(p-3)^5} + \frac{1}{80}$ **15** $\tan^{-1} e^x + c$

16 $x(1 - x^2)^{\frac{1}{2}} + \sin^{-1} x + c$ **17** $\frac{2\sqrt{3}}{3} \tan^{-1} \left(\frac{\tan \frac{x}{2}}{\sqrt{3}} \right) + c$ **18** $-\frac{1}{4} \cot \left(\sin^{-1} \frac{x}{2} \right) + c$ **19** $\frac{3}{2} \tan^{-1} x^2 + c$

20 $\sqrt{x-2} + \frac{\sqrt{2}}{2} \tan^{-1} \sqrt{\frac{x-2}{2}} + c$ **21** $\frac{-1}{x} (\ln|x| + 1) + c$ **22** $\frac{e^{3x}}{9}(3x-1) + c$ **23** $x \sin \left(x + \frac{\pi}{6} \right) + \cos \left(x + \frac{\pi}{6} \right) + c$

24 $\frac{e^{-2x}}{4}(\sin 2x - \cos 2x) + c$ **25** $-2x^2 \cos \frac{x}{2} + 8x \sin \frac{x}{2} + 16 \cos \frac{x}{2} + c$ **26** $x \ln|2x+1| - x + \frac{1}{2} \ln|2x+1| + c$ **27** $x \tan^{-1} \frac{1}{x} + \frac{1}{2} \ln|x^2+1| + c$

28 $\frac{e^{ax}}{a^2+4}(a \sin 2x - 2 \cos 2x) + c$ **29** $\frac{1}{3} \sin^3 x + c$ **30** $-\frac{1}{3} \cos^3 x + c$ **31** $\frac{1}{4} \tan^4 x + c$ **32** $\frac{\theta}{2} + \frac{\sin 2\theta}{4} + c$ **33** $\frac{x}{2} - \frac{\sin 6x}{12} + c$

34 $\frac{\sin 2x}{2} - \frac{1}{6} \sin^3 2x + c$ **35** $-4 \cos \frac{x}{4} + \frac{8}{3} \cos^3 \frac{x}{4} - \frac{4}{5} \cos^5 \frac{x}{4} + c$ **36** $\frac{2}{3} \tan^3 \frac{x}{2} - 2 \tan \frac{x}{2} - x + c$ **37** $\frac{1}{4} \left(x - \frac{\sin 4ax}{4a} \right) + c$

38 $\frac{1}{3} \tan^3 x + \frac{1}{5} \tan^5 x + c$ **39** $\frac{3}{16} \left(2x + 4 \sin \frac{x}{3} + \sin \frac{2x}{3} \right) + c$ **40** $-2(1 - x^2)^{\frac{1}{2}} - \sin^{-1} x + c$

41 $\ln \left| \frac{3x^2}{2} - 2x + 3 \right| - \frac{2\sqrt{14}}{7} \tan^{-1} \left(\frac{\sqrt{14}}{14}(3x-2) \right) + c$ **42** $\frac{x^3}{3} + \frac{x^2}{2} + x + 2 \ln|x-1| + c$ **43** $\frac{x^3}{4} + \frac{x}{4} + \frac{13 \ln|2x-1|}{8} + c$

Chapter 15 **Exercise 9**

1 $\frac{(x-3)^4}{4} + c$ **2** $\frac{2}{9}(3x-5)^{\frac{3}{2}} + c$ **3** $\frac{1}{4}e^{4x-5} + c$ **4** $\frac{12x^{\frac{2}{3}}}{7} - \frac{192x^{\frac{4}{3}}}{23} + \frac{32x^{\frac{5}{3}}}{3} + c$ **5** $-\operatorname{cosec} 4x + c$ **6** 0.0791

7 $\frac{-e^{-2x}}{4}(2x^2 + 2x + 1) + c$ **8** 0.169 **9** $\ln|x^2+1| - 3 \tan^{-1} x + c$ **10** $\frac{1}{2} \sin^{-1} \frac{2x}{5} + c$ **11** $\frac{-1}{4}(2e^x + 1)^{-2} + c$ **12** $-\frac{1}{b} \ln \left| \frac{a+b \cos p}{a+b} \right|$

13 $\frac{-(1+5x)}{25(2+5x)^2} + c$ **14** $\frac{3}{4} \tan^{-1} \left(\frac{x-3}{4} \right) + c$ **15** $\frac{1}{32}(12 + 8 \sin 2x + \sin 4x) + c$ **16** -0.142 **17** 16.5

18 $\frac{-3x^2}{4} \cos 2x + \frac{3x}{4} \sin 2x + \frac{3}{8} \cos 2x + c$ **19** $\sin^{-1} \left(\frac{x+2}{\sqrt{33}} \right) + c$ **20** $\frac{x}{\ln 4} (\ln x - 1) + c$ **21** $\sin^{-1}(x-1) + c$ **22** $\frac{x^5}{25}(5 \ln 2x - x) + c$

23 2.44 **24** 2.10 **25** $\frac{e^{-3x}}{10}(\sin x - 3 \cos x) + c$ **26** $\frac{\sqrt{x}}{25}(2x^2 + 70) + c$ **27** $\frac{1}{5} \tan^5 x + \frac{1}{7} \tan^7 x + c$ **28** $\frac{1}{2} \sin^{-1} 2x + 2\sqrt{1-4x^2} + c$

29 $\frac{2}{\sqrt{5}} \tan^{-1} \left(\sqrt{5} \tan \frac{x}{2} \right) + c$ **30** 1.44 **31** $\frac{-(1+3a)}{3(a+1)^3} + \frac{1}{3}$ **32** $x \cos^{-1} 2x - \frac{\sqrt{1-4x^2}}{2} + c$ **33** 1 **34** $\frac{\pi a^4}{16}$ **35** -0.0280 **36** 1.50

37 $-\frac{1}{4} \ln|\cos 2x| + c$ **38** 0

Chapter 15 **Exercise 10**

1 0.215 **2** 4.48 **3** 0.148 **5** 9.42 **6** $\frac{1}{4}(4-x^2)^{\frac{1}{2}}$ **7** $0.227a^2$ **10** $\frac{4-e^3}{2}$ **11** $\frac{(2a-1)^{\frac{3}{2}}(3a+1)-4}{15}$ **12** 5.66 **13** 11.7 **14** 3.03

Chapter 15 **Review Exercise**

1 $e - e^k + 1$ **2** $\frac{4}{15} \left(\frac{x+2}{2} \right)^{\frac{3}{2}} \left(\frac{3x-4}{30} \right) + k$ **3** $\frac{(k^2-4)^{\frac{3}{2}}}{3\sqrt{5}}$ **4** $x \arctan x - \frac{1}{2} \ln|1+x^2| + k$ **5** 0.307 **6** $a = 25, b = 2, \sin^{-1} \left(\frac{x-2}{5} \right) + c$

7 $a = 1.07$ **8** $-\frac{e^{-2x}}{4}(2x^2 + 2x + 1) + k$ **9 a** $y = \frac{ex}{2}$ **b** $\frac{1}{10}$ **10** $\frac{1}{2} \arctan \left(\frac{x+3}{2} \right) + k$ **11** $-\frac{a}{b} \ln(3 - b \sin x) + k$ **12** 0.690

13 a (0, 1) is a maximum **b** $y = 0$ **d** $\frac{\pi}{2} - 1$ **14** 1 **15 b i** $\frac{2\pi}{9}$ **ii** $\frac{4\pi}{9}$ **iii** $\frac{6\pi}{9}$ **c** $\frac{n\pi}{9}(n+1)$

Chapter 16 **Exercise 1**

1 $y = \frac{x^3}{3} - \cos x + k$ **2** $y = \frac{(3x-7)^5}{15} + k$ **3** $y = -\frac{2}{3}(1-x^2)^{\frac{3}{2}} + k$ **4** $y = -x \cos x + \sin x + k$

5 $y = -\ln|1 - \sin x| + k$ **6** $y = \frac{3}{2k} e^{\frac{3x}{2}} \left[1 - \frac{3}{2k} \right] + c$ **7** $y = -\frac{1}{3}(1 - 15x^2)^{\frac{1}{2}} + k$ **8** $y = \frac{1}{8}(4x - \sin 4x) + k$ **9** $y = \frac{4}{45}(3x+2)^{\frac{5}{2}} + kx + c$

10 $y = -\ln|\cos x| + kx + c$ **11** $y = \frac{x^4}{24} \ln x - \frac{13x^4}{288} + \frac{kx^2}{2} + cx + d$

12 $y = x \cos x_4 \sin x + \frac{kx^3}{6} + \frac{cx^2}{2} + dx + e$ **13** $y = \frac{1}{2} \ln|4x^2 + 3| - \frac{1}{2} \ln 19$ **14** $y = -\frac{3}{4} \cos \left(4x - \frac{\pi}{4} \right) + 2$

15 $y = \frac{(2x-1)^6}{120} + 2x + \frac{239}{240}$ **16** $y = 2x \tan^{-1} x - \ln|1+x^2| + x + 5$

Chapter 16 **Exercise 2**

1 $\frac{y^2}{2} = -\ln|\cos x| + k$ **2** $\tan^{-1} y = \frac{1}{2} \ln|x| + k$ **3** $\frac{1}{2} \ln|3+2y| = -\frac{1}{3} \ln|4-3x| + k$ **4** $y = \ln|\cos x + \sin x| + k$

5 $y^4 = 2x^2 \ln x - x^2 + k$ **6** $e^{-y} = -\left(\frac{5}{6} \ln x - k \right)$ **7** $\ln y = 4 \sin^{-1} \frac{x}{2} + k$ **8** $s^3 = 3(t \sin^{-1} t + (1-t^2)^{\frac{1}{2}} + k)$

9 $3y^3 = \frac{(3x-1)^{11}}{11} + \frac{(3x-1)^2}{2} + k$ **10** $v^2 = \frac{\sin 2at}{2a} + t + k$ **11** $y = \ln \left(\frac{-e^{-2x}}{2} + k \right)$ **12** $\ln|y| = \frac{3}{2} \ln|x^2 + 2x| + k$

13 $\ln y = \frac{-(3-x)^5}{5} + \ln 4 + \frac{1}{5}$ **14** $\frac{3}{2} y^{\frac{3}{2}} = \frac{-e^{-2x}}{2} + \frac{3}{2} + \frac{1}{2e^2}$ **15** $-\cot y = \ln x - 1 - \ln 4$

16 $\frac{1}{2} \ln|2y^2 + 3| = \frac{3}{8} \ln|4x^2 - 1| + \frac{1}{2} \ln 131 - \frac{3}{8} \ln 3$ **17** $\frac{\theta^3}{3} = \frac{e^{2t}}{5}(2 \sin t - \cos t) + \frac{\pi^3}{24} + \frac{1}{5}$ **18** $\frac{1}{3} \sin^{-1} 3s = \frac{t^2}{2} - \frac{1}{2}$

Chapter 16 **Exercise 3**

1 $y = -\ln|\cos x| + k$ **2** $x = \frac{\omega^2 t^3}{6} + kt + c$ **3** $x = \frac{-\omega \sin nt}{n^2} + kt + c$ **4** $\frac{y^3}{3} = \ln|1+x^3| + 9$ **5** $\tan y = \tan x + k$

6 a $\frac{dz}{dx} = 1 + \frac{dy}{dx} \tan^{-1} z = x + k$ **6 c** $y = \tan(x+k) - x$ **7** $y = \frac{Bx^4}{24A} - \frac{Bx^3}{12A} + \frac{Bx}{24A}$ **8** $V = -4 \cos \left(t + \frac{\pi}{4} \right) + 6$

9 $r = 4t \ln t - 4t + 24 - 20 \ln 5$

Chapter 16 **Exercise 5**

1 $v = \frac{4t^3}{3} + t$ $s = \frac{t^4}{3} + \frac{t^2}{2}$ **2 a** $v = \frac{-3t^4}{2} + 10$ **2 b** -6134 ms⁻¹ **2 c** $s = \frac{-3t^5}{10} + 10t$ **2 d** -29900 m **3** 26.6m

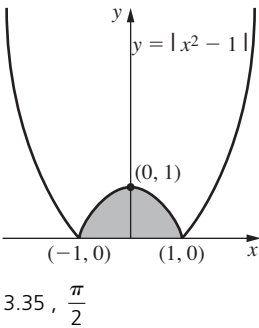
4 a $v = \frac{\sin 3t}{3}, s = -\frac{\cos 3t}{9} + \frac{11}{18}$ **4 b** $\frac{\pi}{3}$ sseconds **5** $s = -\frac{t^2}{2} - t - 2 \ln|l-1| + 14$ **6** $v = \pm \sqrt{2 - 2 \cos \left(s + \frac{\pi}{4} \right)}$

7 $v = \pm \sqrt{\frac{2se^{2s} - e^{2s} + 9}{2}}$ **8** $v = \pm \sqrt{\frac{(2s-1)^5 + 223}{5}}$ **9 a** $\frac{7}{4}$ ms⁻¹ **10 b** $\frac{e^8 - 137}{4}$ **11 a** 2.60 **11 b** $v = 220e^{-2.60t}$ **c** 108m

12 a $v = \frac{2}{\omega}$ **12 b** $\frac{2\pi}{\omega}$ **13** $v = \frac{g}{k} - \frac{g}{ke^{kt}}, \text{ Yes } v \rightarrow \frac{g}{k}$

Chapter 16 Exercise 6

- 1 a 176 1 b 107 1 c 57.4 1 d 8.90 1 e 104 1 f 2060 1 g 327 1 h 1.23 1 i 1.08 1 j 274 2 a 8π 2 b 5.98 2 c 0.592
- 2 d 0.622 2 e 113 2 f 0.965 2 g 12.8 3 $\frac{2\pi a^5}{5}$ 4 145 5 1940 6 84.2 7 5cm, 196 cm² 8
- 9 $\frac{49\pi}{16}$ 10 $\pi^2 b^2$ 11 $-4n \cos(n+1)\pi$
- 12 a $\frac{1}{4}(2 \sin^{-1} x + 2x\sqrt{1-x^2}) + k$ b. $\frac{\pi}{4}(\pi - 2 \sin^{-1} a + 2a\sqrt{1-a^2})$



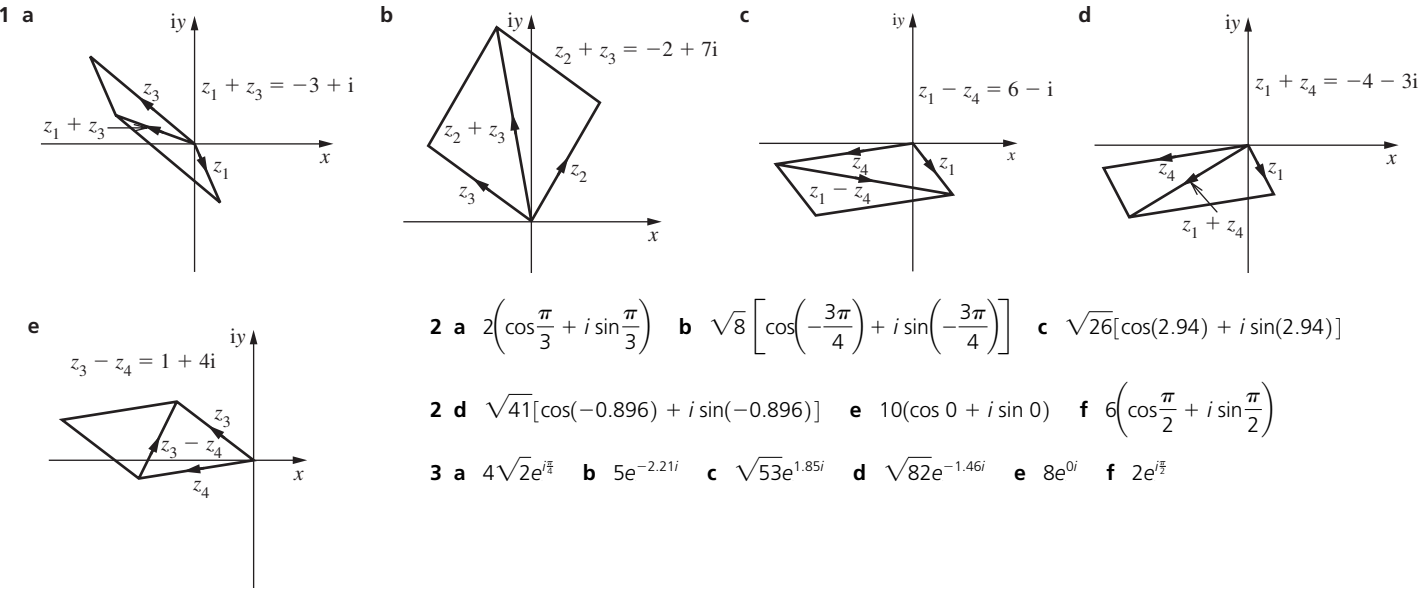
Chapter 17 Exercise 1

- 1 a 18i b 38i c 112i d 60i 2 a 12i b 15i c -24i d -52i 3 a 240i b -32 c 45i d 96i e -72i f 35 g -90i
- 4 a $-\frac{15}{2}$ b 2 c $\frac{5}{2}i$ d -16i 5 a 9 b -1 6 a 9i b $-\frac{5}{3} + \frac{7i}{3}$ c $-\frac{1}{2}i$ d 1 e $1 - \frac{3}{4}i$

Chapter 17 Exercise 2

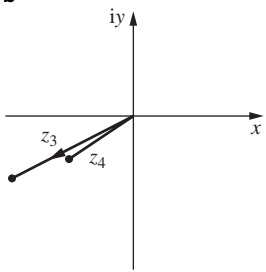
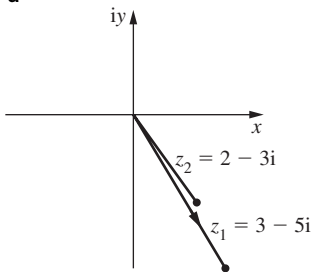
- 1 a $8 + 16i$ b $18 + 28i$ c $7 + i$ d $14 - 21i$ e $-7 + 11i$ 2 a $3 - 5i$ b $13 + 2i$ c $-20 + 10i$ d $7 + 13i$
- 3 a $-4 + 19i$ b $47 + 35i$ c $7 + 2i$ d $115 - 111i$ e 306 f $a^2 + b^2$ g $1 + 70i$ h $117 + 44i$ i $x^2 - y^2 + 2ixy$
- 3 j $m(m^2 - 3n^2) + i(3m - n)$ k $(-m^3 - 18m^2 + 12m + 24) + i(3m^3 - 6m^2 - 36m + 8)$ 4 a $\frac{3+i}{5}$ b $\frac{-15+10i}{13}$ c $\frac{7+26i}{29}$
- 4 d $\frac{-10-33i}{29}$ e $6 - 2i$ f $\frac{2x^2 - y^2 + 3ixy}{4x^2 + y^2}$ g $\frac{3+4i}{5}$ h $-\frac{3}{2} + \frac{i}{2}$ i $\frac{(2x-5y) + i(5x+2y)}{x^2 + y^2}$ j $\frac{x(y^2 + 3x^2 + 2ixy)}{x^2 + y^2}$
- 5 a -4 b $-11 - 2i$ c $-237 - 3116i$ d $-8432 - 5376i$ 6 a $x = 15, y = -7$ b $x = 8, y = 0$ c $x = 0, y = -3$
- 6 d $x = 5, y = 12$ e $x = -6, y = -3$ f $x = -\frac{5}{17}, y = -\frac{14}{17}$ g $x = -21, y = 20$ h $x = y = \pm\sqrt{\frac{15}{2}}$ i $x = -\frac{72}{25}, y = \frac{29}{25}$
- 6 j $x = -3, y = 1$ k $x = -\frac{33}{169}, y = \frac{2591}{169}$ 7 a $\operatorname{Re}(z) = 27, \operatorname{Im}(z) = -8$ b $\operatorname{Re}(z) = -\frac{53}{185}, \operatorname{Im}(z) = -\frac{89}{185}$ c $\operatorname{Re}(z) = \frac{117}{145}, \operatorname{Im}(z) = \frac{41}{145}$
- 7 d $\operatorname{Re}(z) = \frac{72}{65}, \operatorname{Im}(z) = -\frac{61}{65}$ e $\operatorname{Re}(z) = \frac{2a}{4+b^2} - \frac{12}{16+a^2}, \operatorname{Im}(z) = -\frac{ab}{4+b^2} - \frac{3a}{16+a^2}$ f $\operatorname{Re}(z) = 0, \operatorname{Im}(z) = -\frac{2xy}{x^2 + y^2}$
- 7 g $\operatorname{Re}(z) = -597, \operatorname{Im}(z) = 122$ h $\operatorname{Re}(z) = \cos\frac{2\pi}{3}, \operatorname{Im}(z) = \sin\frac{2\pi}{3}$ i $\operatorname{Re}(z) = -128, \operatorname{Im}(z) = -128\sqrt{3}$
- 7 j $\operatorname{Re}(z) = \frac{x}{1+y^2} + \frac{12}{25}, \operatorname{Im}(z) = -\frac{xy}{1+y^2} + \frac{9x}{25}$ 8 a $1 + 4i, -1 - 4i$ b $1.10 + 0.455i, -1.10 - 0.455i$ c $2.12 - 0.707i, -2.12 + 0.707i$
- 8 d $3.85 + 1.69i, -3.85 - 1.69i$ e $1.92 + 1.30i, -1.92 - 1.30i$ f $0.734 - 0.454i, -0.734 + 0.454i$
- 8 g $0.704 - 0.369i, -0.704 + 0.369i$ h $1.59 + 1.42i, -1.59 - 1.42i$ i $0.541 + 0.0416i, -0.541 - 0.0416i$ 9 a $x = -3 \pm i$
- 9 b $x = \frac{-1 \pm i\sqrt{3}}{2}$ c $x = \frac{-3 \pm i\sqrt{51}}{2}$ d $x = \frac{-3 \pm i\sqrt{6}}{3}$ e $x = \frac{-3 \pm i\sqrt{95}}{4}$ 10 a $x^2 - 4x + 13 = 0$ b $x^2 - 6x + 10 = 0$
- 10 c $x^2 - 8x + 25 = 0$ d $x^3 - 3x^2 + 7x - 5 = 0$ e $x^3 - 8x^2 + 25x - 26 = 0$ f $x^4 - 10x^3 + 20x^2 + 90x - 261 = 0$
- 11 a $x^2 - 4x + 53 = 0$ b $x^2 - 8x + 25 = 0$ c $x^2 - 14x + 85 = 0$ d $x^2 - 2ax + a^2 + b^2 = 0$ 12 $x^4 - 10x^3 + 42x^2 - 82x + 65 = 0$
- 13 3, $1 - i$ 14 a 3, $-1 + 2i, -1 - 2i$ b 2, $-1 + 3i, -1 - 3i$ 15 $\frac{-214 + 735i}{53}$ 16 $-8 + 2i, -8 - i$ 17 $\frac{348 - 115i}{13}$
- 18 $z_1 = \frac{26-2i}{17}, z_2 = \frac{21+i}{17}$ 19 $\frac{3-i\sqrt{3}}{2}$ 20 $p = -\frac{3}{5}, q = \frac{9}{5}$ 21 $\frac{88-966i}{25}$

Chapter 17 Exercise 3



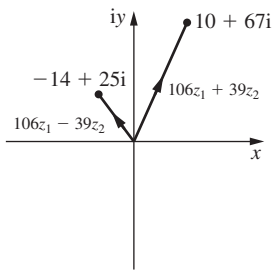
- 2 a $2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ b $\sqrt{8}\left[\cos\left(-\frac{3\pi}{4}\right) + i\sin\left(-\frac{3\pi}{4}\right)\right]$ c $\sqrt{26}[\cos(2.94) + i\sin(2.94)]$
- 2 d $\sqrt{41}[\cos(-0.896) + i\sin(-0.896)]$ e $10(\cos 0 + i\sin 0)$ f $6\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$
- 3 a $4\sqrt{2}e^{i\frac{\pi}{4}}$ b $5e^{-2.21i}$ c $\sqrt{53}e^{1.85i}$ d $\sqrt{82}e^{-1.46i}$ e $8e^{0i}$ f $2e^{i\frac{\pi}{2}}$

- 4 a $1 + i\sqrt{3}$ b $-\frac{\sqrt{15}}{2} + i\frac{\sqrt{5}}{2}$ c $5\sqrt{2} - 5i\sqrt{2}$ d $3.74 - 1.00i$ e $-\frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i$ f $-\frac{\sqrt{5}}{2} + i\frac{\sqrt{15}}{2}$ g $\frac{15\sqrt{3}}{2} - \frac{15}{2}i$
- 4 h $1.67 - 4.03i$ 5 a $r = \sqrt{313}, \theta = 0.825$ b $r = \sqrt{701}, \theta = 1.38$ c $r = \sqrt{370}, \theta = 0.487$ d $r = 4\frac{\sqrt{370}}{5}, \theta = 1.41$
- 6 a Root 1 $r = 2.65, \theta = 2.17$ Root 2 $r = 2.65, \theta = -2.17$ b Root 1 $r = \sqrt{5}, \theta = 1.11$ Root 2 $r = \sqrt{5}, \theta = -1.11$
- 6 c Root 1 $r = \sqrt{7}, \theta = 0.714$ Root 2 $r = \sqrt{7}, \theta = -0.714$ 7 a i $z_1 = 13e^{-1.18i}$ ii $z_2 = 5e^{2.21i}$ iii $z_3 = 25e^{-0.284i}$ iv $z_4 = 2e^{i\frac{\pi}{3}}$
- 7 b i $r = 65, \theta = 1.04$ ii $r = 325, \theta = -1.46$ iii $r = 26, \theta = -0.129$ iv $r = 2.5, \theta = 1.17$ v $r = 5, \theta = -2.50$ vi $r = 7.5, \theta = -2.22$
- 7 vii $r = 2.5, \theta = -1.33$ viii $r = 150, \theta = 0.763$
- 8 a and b c Rotation of $\frac{\pi}{2}$ radians clockwise.

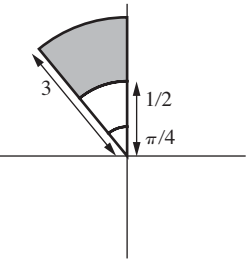


- 9 a $z_1 = \sqrt{7}[\cos(-0.714) + i\sin(-0.714)]z_2 = \sqrt{2}\left[\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right]$
- 9 b $|z_1z_2| = \sqrt{14}, \arg(z_1z_2) = -3.07, \left|\frac{z_1}{z_2}\right| = \sqrt{\frac{7}{2}}, \arg\left(\frac{z_1}{z_2}\right) = 1.64$

- 10 $z_1 = -\frac{1}{53} + \frac{23}{53}i, z_2 = \frac{4}{13} + \frac{7}{13}i$ 11 b $z = 1 + 0i, \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}$ 12 b i 16 ii 10



- 14 a $1[\cos(0.841) + i\sin(0.841)], 1[\cos(-0.841) + i\sin(-0.841)]$ b 1.68 radians
- 15 a $r = \frac{3}{2}, \theta = 1.23$ 15 b c No.



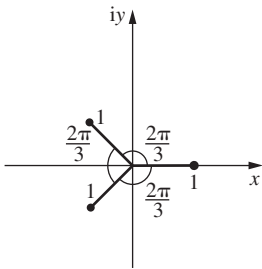
Chapter 17 Exercise 4

- 1 a** $1024(\cos 10\theta + i \sin 10\theta)$ **b** $\cos 25\theta + i \sin 25\theta$ **c** $\frac{1}{243}[\cos(-5\theta) + i \sin(-5\theta)]$ **d** $\cos(-9\theta) + i \sin(-9\theta)$ **e** $\cos \frac{1}{2}\theta + i \sin \frac{1}{2}\theta$
- 1 f** $\frac{1}{\sqrt[3]{4}}\left[\cos\left(-\frac{1}{3}\theta\right) + i \sin\left(-\frac{1}{3}\theta\right)\right]$ **g** $\cos 0 + i \sin 0$ **h** $\cos\left(-\frac{\pi}{2}\right) + i \sin\left(-\frac{\pi}{2}\right)$ **i** $\cos\left(\frac{3\pi}{4}\right) + i \sin\left(\frac{3\pi}{4}\right)$ **j** $\cos\left(\frac{\pi}{10}\right) + i \sin\left(\frac{\pi}{10}\right)$
- 2 a** $(\cos \theta + i \sin \theta)^7$ **b** $4(\cos \theta + i \sin \theta)^{\frac{1}{2}}$ **c** $6(\cos \theta + i \sin \theta)^{-3}$ **d** $(\cos \theta + i \sin \theta)^{-\frac{1}{2}}$ **e** $(\cos \theta + i \sin \theta)^{-2}$ **f** $(\cos \theta + i \sin \theta)^{-\frac{1}{2}}$
- 3 a** $\cos 8\theta + i \sin 8\theta$ **b** $\cos \frac{5}{2}\theta + i \sin \frac{5}{2}\theta$ **c** $\cos 3\theta + i \sin 3\theta$ **d** $\cos \theta - i \sin \theta$ **e** $\cos 11\theta + i \sin 11\theta$ **f** $\cos 3\theta - i \sin 3\theta$
- 3 g** $\cos \frac{1}{6}\theta + i \sin \frac{1}{6}\theta$ **h** $\cos \frac{3}{4}\pi - i \sin \frac{3}{4}\pi$ **i** $\cos \frac{3}{4}\pi + i \sin \frac{3}{4}\pi$ **j** $\cos \frac{\pi}{12} + i \sin \frac{\pi}{12}$ **4 a** $2 + 3i, -2 - 3i$ **b** $0.644 - 1.55i, -0.644 + 1.55i$
- 4 c** $2^{\frac{1}{2}}\left[\cos\left(-\frac{3\pi}{4}\right) + i \sin\left(-\frac{3\pi}{4}\right)\right], 2^{\frac{1}{2}}\left[\cos\left(-\frac{\pi}{12}\right) + i \sin\left(-\frac{\pi}{12}\right)\right], 2^{\frac{1}{2}}\left[\cos\left(\frac{7\pi}{12}\right) + i \sin\left(\frac{7\pi}{12}\right)\right]$ **d** $1.69 - 0.606i, -0.322 + 1.77i, -1.37 - 1.16i$
- 4 e** $1.45 + 0.354i, -0.354 + 1.45i, 0.354 - 1.45i, -1.45 - 0.354i$ **f** $1.54 - 0.640i, 1.09 + 1.27i, -0.872 + 1.42i$

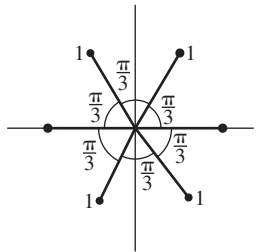
4 f $-0.132 - 1.67i, -1.62 - 0.389i$ **g** $2^{\frac{1}{2}}\left[\cos\left(-\frac{35\pi}{36}\right) + i \sin\left(-\frac{35\pi}{36}\right)\right], 2^{\frac{1}{2}}\left[\cos\left(-\frac{23\pi}{36}\right) + i \sin\left(-\frac{23\pi}{36}\right)\right], 2^{\frac{1}{2}}\left[\cos\left(-\frac{11\pi}{36}\right) + i \sin\left(-\frac{11\pi}{36}\right)\right]$

4 g $2^{\frac{1}{2}}\left[\cos\left(\frac{\pi}{36}\right) + i \sin\left(\frac{\pi}{36}\right)\right], 2^{\frac{1}{2}}\left[\cos\left(\frac{13\pi}{36}\right) + i \sin\left(\frac{13\pi}{36}\right)\right], 2^{\frac{1}{2}}\left[\cos\left(\frac{25\pi}{36}\right) + i \sin\left(\frac{25\pi}{36}\right)\right]$

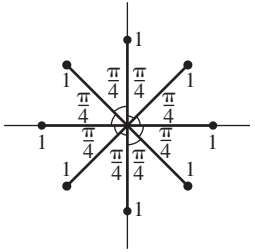
5 a



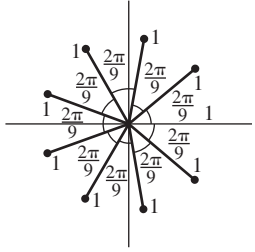
b



c



d



6 a $16\left[\cos\left(\frac{\pi}{2}\right) + i \sin\left(\frac{\pi}{2}\right)\right]$ **b** $1.85 + 0.765i, -0.765 + 1.85i, 0.765 - 1.85i, -1.85 - 0.765i$ **7 a** $2\left[\cos\left(\frac{\pi}{3}\right) + i \sin\left(\frac{\pi}{3}\right)\right]$

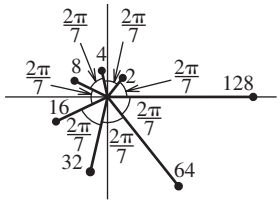
7 b Real part is 2^{15} . Imaginary part is $-2^{15}\sqrt{3}$ **9 b** $\tan\left(-\frac{15\pi}{16}\right), \tan\left(-\frac{7\pi}{16}\right), \tan\left(\frac{\pi}{16}\right), \tan\left(\frac{9\pi}{16}\right)$ **10** $-\frac{1}{2} - i\frac{\sqrt{3}}{2}$

11 a $z_2 = 5[\cos(-0.927) + i \sin(-0.927)]$ **b** $\frac{4}{25}$ **12** Product = 4, Sum = -2

15 a $z_1 = 2\left(\cos \frac{2\pi}{7} + i \sin \frac{2\pi}{7}\right)$ **b** $z_1^2 = 4\left(\cos \frac{4\pi}{7} + i \sin \frac{4\pi}{7}\right), z_1^3 = 8\left(\cos \frac{6\pi}{7} + i \sin \frac{6\pi}{7}\right)$

$z_1^4 = 16\left[\cos\left(-\frac{6\pi}{7}\right) + i \sin\left(-\frac{6\pi}{7}\right)\right], z_1^5 = 32\left[\cos\left(-\frac{4\pi}{7}\right) + i \sin\left(-\frac{4\pi}{7}\right)\right], z_1^6 = 64\left[\cos\left(-\frac{2\pi}{7}\right) + i \sin\left(-\frac{2\pi}{7}\right)\right], z_1^7 = 128(\cos 0 + i \sin 0)$

15 c



15 d Rotate $\frac{2\pi}{7}$ anticlockwise. Enlargement scale factor 2.

16 b $\cos\left(\frac{\pi}{3}\right) + i \sin\left(\frac{\pi}{3}\right), \cos\left(-\frac{\pi}{3}\right) + i \sin\left(-\frac{\pi}{3}\right)$ **c** $\cos\left(-\frac{2\pi}{3}\right) + i \sin\left(-\frac{2\pi}{3}\right)$

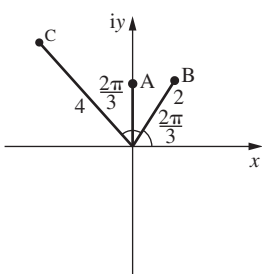
17 c $z^6 + 6z^4 + 15z^2 + 20 + \frac{15}{z^2} + \frac{6}{z^4} + \frac{1}{z^6}$ **d** $a = \frac{1}{32}, b = \frac{3}{16}, c = \frac{15}{32}, d = \frac{5}{16}$

Chapter 17 Review Exercise

1 $r = \sqrt{14.8}, \theta = 2.06$ **2** $k = -2$ **3** $a = -2, b = 5$ **4** 4 **5 b** 4294967296 **6** $x = -\frac{47}{65}, y = -\frac{1}{65}$ **7** $x^3 - 5x^2 + 10x - 12 = 0$

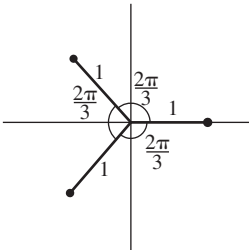
8 Real part $= \frac{x^3 + xy^2 + x}{x^2 + y^2}$, Imaginary part $= \frac{x^2y + y^3 - y}{x^2 + y^2}$ **9** $\sqrt{3}$ **10 a** $|z| = 2, \arg(z) = \frac{\pi}{3}$ **b** $|z^2| = 4, \arg(z^2) = \frac{2\pi}{3}$

10 c i



ii $2\sqrt{3}$ **iii** $2\sqrt{3} - \frac{3}{2}$ **11 b** $1, \frac{-1 + i\sqrt{3}}{2}, \frac{-1 - i\sqrt{3}}{2}$

11 c $\cos 0 + i \sin 0, \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}, \cos\left(-\frac{2\pi}{3}\right) + i \sin\left(-\frac{2\pi}{3}\right)$



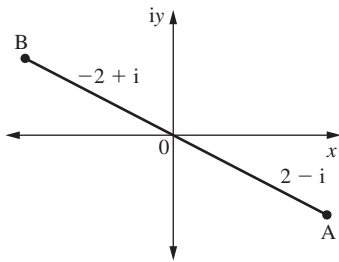
11 d Each side has length $\sqrt{3}$. Area of triangle $= \frac{3\sqrt{3}}{4}$ **12 a** $a = -\frac{5}{3}, b = \frac{16}{9}$ **b** $a = -\frac{17}{4} - \frac{31i}{4}, b = -\frac{17}{4} + \frac{31i}{4}$

13 d i $a = 32, b = -32, c = 6$ **ii** 6 **14** $z = 5 + i, \omega = 6 - i$ **15 a i** $\cos^3 \theta - 3 \cos \theta \sin^2 \theta + i(3 \cos^2 \theta \sin \theta - \sin^3 \theta)$ **c** $\frac{23\sqrt{2}}{20}$

16 a i 1 **ii** $\frac{2\pi}{3}$ **c** $\frac{3}{2} + \frac{3\sqrt{3}}{2}i$ **17** $|z| \leq 5, \frac{\pi}{3} \leq \arg(z) \leq 1.98, -2 \leq \operatorname{Re}(z) \leq \frac{5}{2}$ **18 a** $\frac{10k + i(k^2 - 21)}{k^2 + 49}$ **b** $k = \pm\sqrt{21}$

19 a $1, \frac{-1 + i\sqrt{3}}{2}, \frac{-1 - i\sqrt{3}}{2}$ **c** $\begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ **d** $x = -1, y = z = 2$ **21 a** $2 - i, -2 + i$ **b**

21 c $(\sqrt{3}, 2\sqrt{3}), (-\sqrt{3}, -2\sqrt{3})$



Answers are given when asked to form a conjecture

Chapter 18 Exercise 3

1 $D^n = \begin{pmatrix} 1 & 2^n - 1 \\ 0 & 2^n \end{pmatrix}$ **2** $\sum_{r=1}^n 3r + 2 = \frac{1}{2}n(7n + 3)$ **3** $\sum_{r=1}^n 4r - 7 = n(2n - 5)$ **4** Any value $\geq 21p$

5 sum of the first n odd numbers $= n^2$ **6** $n^2 + 2n + 2$ **7** $n^2 + 4$

Chapter 18 Review Exercise

8 $(n + 1)! - 1$ **16** $M^n = \begin{pmatrix} n + 1 & -n \\ n & 1 - n \end{pmatrix}$

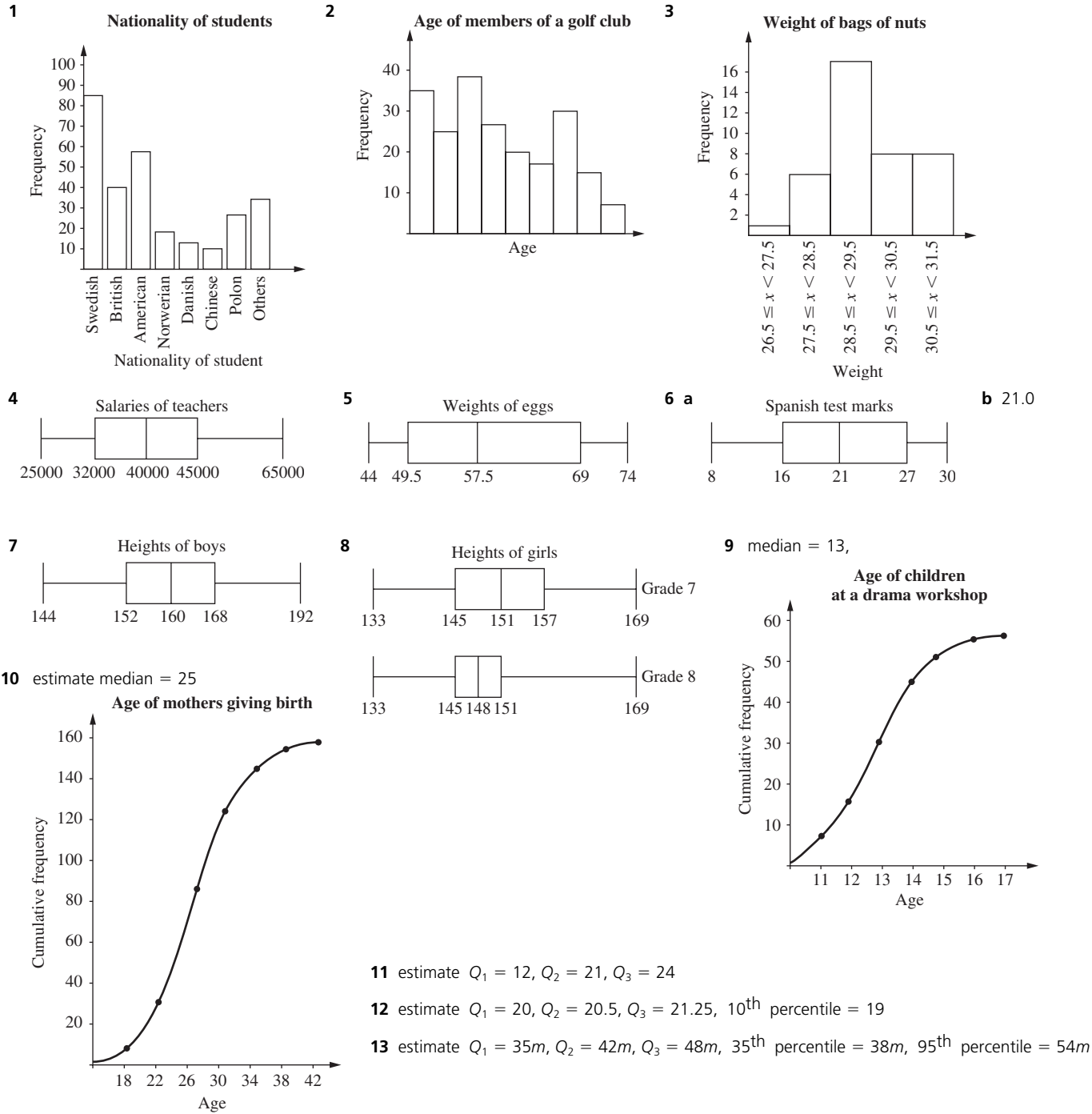
Chapter 19 Exercise 1

1 a Continuous **b** Discrete **c** Continuous **d** Continuous **2** mode = 4., median = 4, mean = 4.15 **3** mode = Blue

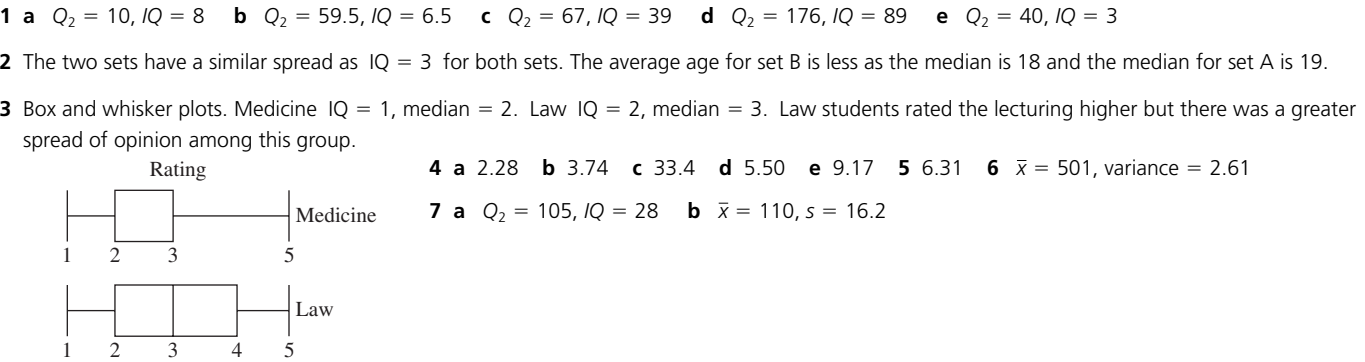
4 b 81-90 **c** 78.2 **5** 16.7 **6 a** 1.58m. **b** There is no information about the ages or gender of the students. **7** 0.927

8 a People below this height are not allowed on the ride. **b** Mean = 1.79m

Chapter 19 Exercise 2



Chapter 19 Exercise 3



Chapter 19 Exercise 4

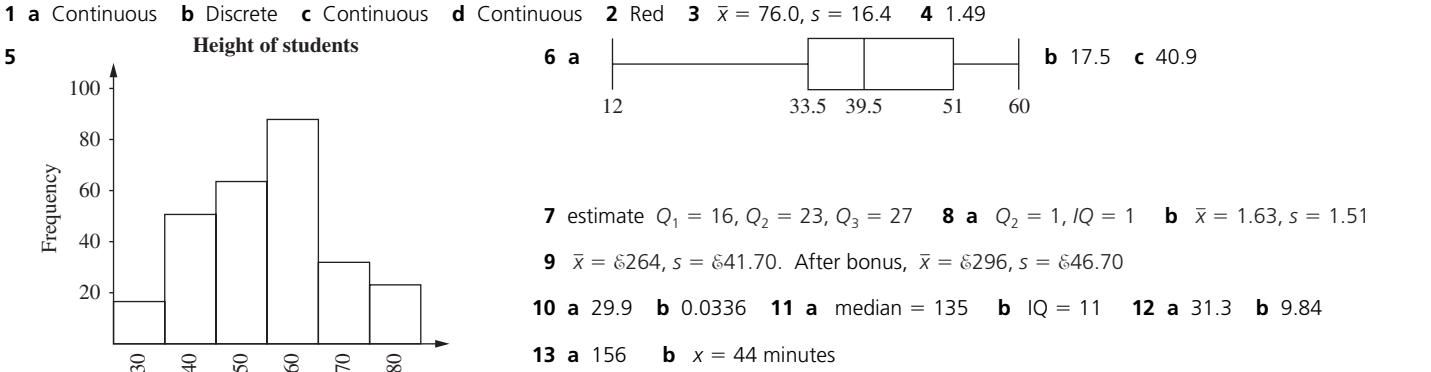
1 **i a** $Q_1 = 8.3, Q_2 = 9.9, Q_3 = 11.9$ **b** $\bar{x} = 9.8, s = 1.79$ **ii a** $Q_1 = 183, Q_2 = 263, Q_3 = 298$ **b** $\bar{x} = 238, s = 60.2$

1 **iii a** $Q_1 = 34000, Q_2 = 45500, Q_3 = 57250$ **b** $\bar{x} = 44500, s = 14300$ **iv a** $Q_1 = 0.62, Q_2 = 0.755, Q_3 = 0.845$ **b** $\bar{x} = 0.724, s = 0.189$

2 Graph $IQ = 3$ **3** Daniel Graph, median = 185.5, range = 167 Paul Graph, median = 198.5, range = 71

4 $\bar{x} = 40.2, s = 29.1$, New mean = 46.2, $s = 33.5$ **5** $\bar{x} = 16.6, s = 1.44$

Chapter 19 Review Exercise



Chapter 20 Exercise 1

1 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$ **2 a** $\frac{1}{10}$ **b** $\frac{1}{2}$ **c** $\frac{3}{10}$ **d** $\frac{2}{5}$ **3 a** $\frac{1}{4}$ **b** $\frac{13}{20}$ **c** $\frac{9}{10}$ **d** $\frac{13}{20}$ **e** 0 **4 a** 0.48

4 **b** Because the probability of either a novel or a mathematics book is 1. **5** No. $P(A \cup B) \neq 1$ **6 a** $\frac{7}{11}$ **b** $\frac{922}{1155}$ **7** $\frac{1}{8}$ **8 a** $\frac{1}{5}$ **b** $\frac{9}{15}$

7 **c** $\frac{13}{15}$ **d** $\frac{13}{15}$ **e** $\frac{2}{3}$ **9 a** $\frac{1}{4}$ **b** $\frac{5}{8}$ **c** $\frac{1}{8}$ **d** 1 **e** $\frac{3}{4}$ **f** $\frac{1}{2}$ **g** 0 **10 a** $\frac{1}{9}$ **b** $\frac{5}{6}$ **c** $\frac{1}{6}$ **d** $\frac{1}{6}$ **e** $\frac{1}{9}$ **f** $\frac{1}{4}$ **g** $\frac{2}{3}$

11 **a** 0.1 **b** 1 **c** 0.5 **12** $\frac{1}{10}$ **13** $\frac{1}{4}$ **14** 0.72 **15 a** $\frac{1}{3}$ **b** $\frac{13}{18}$ **c** $\frac{1}{4}$ **d** $\frac{4}{9}$ **e** $\frac{1}{3}$

15 **f** Because it is not possible to have one die showing a 5 and for the sum to be less than 4. **16** $\frac{23}{32}$

17 **a** $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** 1 The events are mutually exclusive because it is not possible to have an even number that is prime.

18 **a** Events X and Y are not mutually exclusive because 2 fish of type A and 2 fish of type B fit both.

18 **b** Events X and Z are not mutually exclusive because 2 fish of type A, 1 fish of type B and 1 fish of type C fit both. **c** Events Y and Z are mutually exclusive because the event Y does not allow a fish of type C and event Z does.

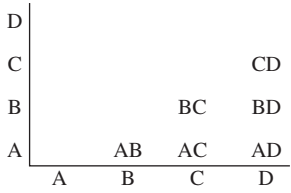
19 0.15 **20 a** $\frac{83}{500}$ **b** $\frac{1}{25}$

Chapter 20 Exercise 2

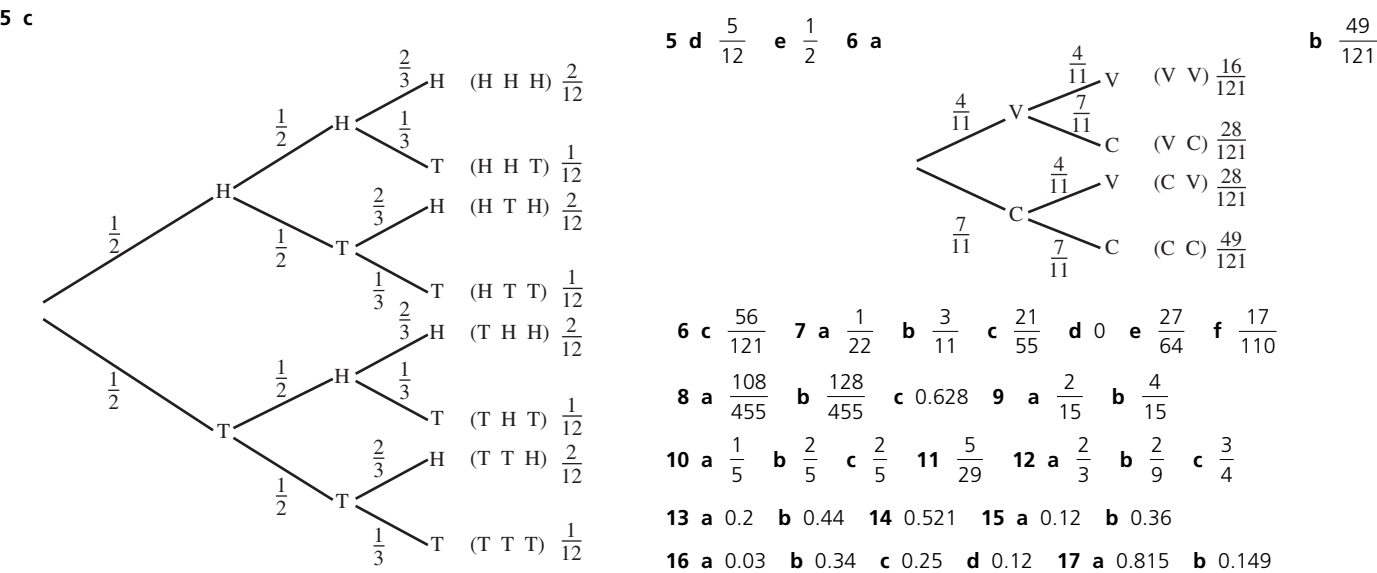
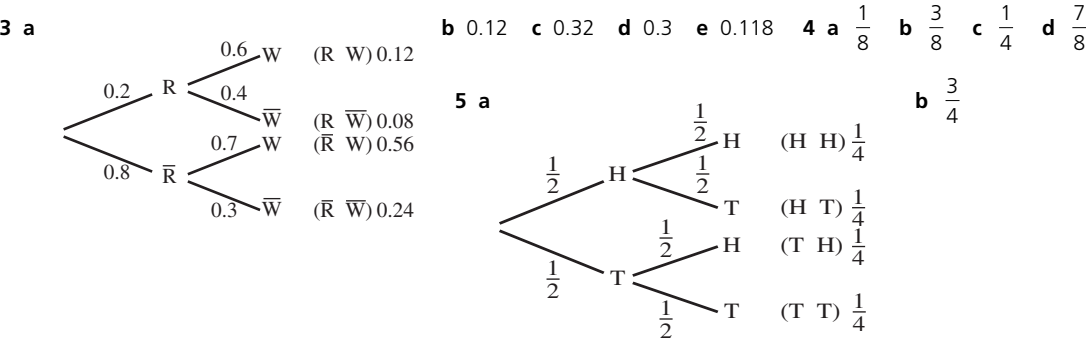
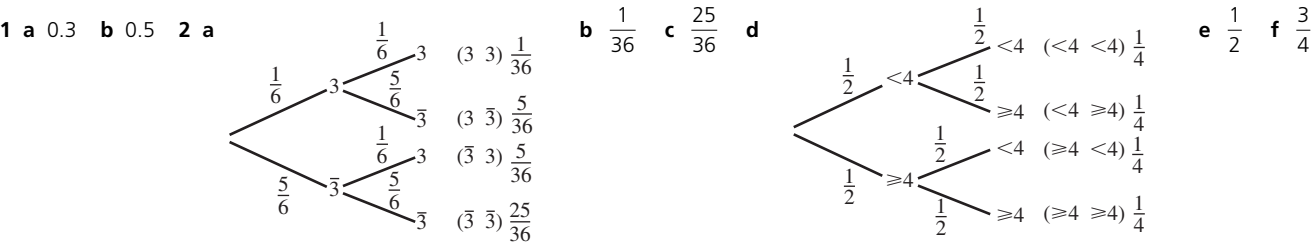
1 **a** $\frac{5}{42}$ **b** $\frac{3}{7}$ **2 a** $\frac{9}{15}$ **b** $\frac{4}{21}$ **c** $\frac{1}{2}$ **3 a** $\frac{7}{16}$ **b** $\frac{1}{4}$ **c** $\frac{2}{7}$ **d** $\frac{1}{2}$

4 **a** $\frac{1}{14}$ **b** $\frac{1}{6}$ **c** $\frac{2}{7}$ **5 a** $\frac{2}{5}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$ **6** 0.0768 **7 a** $\frac{181}{208}$

7 **b** No, because $P(A \cup B) \neq 1$ 8 $\frac{14}{17}$ 9 **a** **b** $\frac{2}{3}$ **c** $\frac{1}{2}$



Chapter 20 Exercise 3



Chapter 20 Exercise 4

1 **a** 0.974 **b** 6 **c** 6 2 **a** 0.832 **b** 24 3 0.821 4 $\frac{3}{4}$ 5 $\frac{2}{3}$ 6 $\frac{45}{53}$ 7 $\frac{8}{11}$ 8 **a** i $\frac{11}{609}$ ii 0.0212 iii 0.0153 **b** 0.0839

9 0.84 10 **a** 0.222 **b** 0.074 **c** 0.144 **d** 7 11 **a** 0.00103 **b** 0.479 **c** 0.524 **d** 0.561 12 **a** 0.3 **b** 0.35 **c** 0.075 **d** 0.3

13 **a** $\frac{1}{64}$ **b** $\frac{1}{64}$ **c** 19 14 **a** 0.0191 **b** 0.00459 **c** 0.164 15 **a** $\frac{1}{27000}$ **b** 90 **c** $\frac{1}{900}$

Chapter 20 Exercise 5

1 720 2 16065 3 24 4 60 5 10080 6 604800 7 241920 8 201600 9 **a** 36 **b** 6 **c** 12 **d** 24 10 70 11 **a** 90720 **b** 5040

11 **c** 2520 12 **a** 40320 **b** 5040 **c** 2520 13 **a** 4989600 **b** 1270080 14 **a** 5040 **b** 4320 **c** 720 15 **a** 42 **b** 10 16 **a** 9000000

16 **b** Increases by 16000000 17 **a** 831600 **b** 176400 **c** 151200 18 119 19 1 20 **a** 4 **b** 16 **c** 24 **d** 62 21 **a** 60 **b** 30

22 **a** 28 **b** 28 **c** 35 23 **a** 120 **b** 90 24 756756 25 70 26 210 27 3185325 28 33

Chapter 20 Exercise 6

1 **a** 100 **b** $\frac{3}{4}$ **c** $\frac{1}{5}$ **d** $\frac{1}{5}$ 2 **a** 362880 **b** i $\frac{1}{9}$ ii $\frac{8}{9}$ **c** $\frac{1}{189}$ 3 **a** 15 **b** $\frac{1}{15}$ 4 **a** 360 **b** $\frac{2}{3}$ **c** $\frac{2}{3}$ 5 **a** 3838380

5 **b** $\frac{3}{20}$ **c** $\frac{5}{39}$ 6 **a** 10440 **b** $\frac{12}{29}$ **c** $\frac{1}{10}$ 7 **a** 16 **b** i $\frac{1}{2}$ ii $\frac{3}{8}$ iii $\frac{3}{16}$ 8 **a** 3150 **b** $\frac{1}{30}$ **c** $\frac{1}{45}$ **d** $\frac{1}{6}$ 9 **a** 720 **b** $\frac{1}{15}$

9 **c** $\frac{1}{30}$ 10 **a** 3628800 **b** $\frac{1}{30}$ **c** $\frac{7}{15}$ **d** $\frac{1}{35}$

Chapter 20 Review Exercise

1 **a** $\frac{1}{6}$ **b** Events are not independent since $P(Black) \times P(Brown) = \frac{1}{3}$ and $P(Black \cap Brown) = \frac{1}{6}$

Events are not mutually exclusive because $P(Black \cap Brown) \neq 0$ Events are exhaustive since $P(Black \cup Brown) = 1$

2 **a** 0.995 **b** 3 3 **a** 151200 **b** 10080 4 **a** $\frac{1}{6}$ **b** 0.0670 **c** $\frac{1}{6} \times \left(\frac{5}{6}\right)^{2n-2}$ 5 $\frac{10}{13}$ 6 62 7 **a** 453600

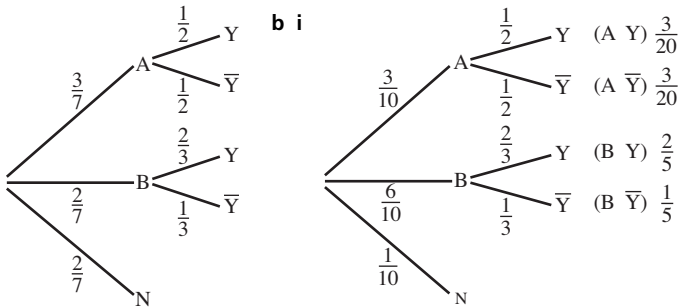
7 **b** 90720 **c** 362880 8 $\frac{19}{30}$ 9 **a** 15 **b** $\frac{8}{15}$ **c** $\frac{4}{15}$ 10 0.888 11 **a** 30240 **b** 30238 **c** 126 **d** 32 12 $\frac{4}{5}$ 13 **a** 360

13 **b** 216 14 0.048 15 **a** 20160 **b** $\frac{1}{4}$ **c** $\frac{1}{56}$ 16 **a** $\frac{17}{42}$ $\frac{9}{17}$

16 **b** ii $\frac{5}{6}$ iii $\frac{7}{25}$ iv $\frac{28}{51}$ 17 **a** 252 **b** 196 **c** 186

18 **a** $\frac{3n+1}{2}$ **b** $\frac{n+1}{3n+1}$ 20 **a** 0.549 **b** 0.369 **c** 0.439

21 **a** 0.581 **b** 0.0918 **c** 0.0663



Chapter 21 Exercise 1

1 **a** $b = 0.2$ **b** 0.6 **c** 0.65 **d** 0.8 **e** 2 2 **a** $a = 0.33$ **b** 0.87 **c** 0.75 **d** 0.73 **e** 0.25 **f** 7

3 **a**

X	0	1	2	3
$P(X = x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

b

X	0	1	2	3
$P(X = x)$	$\frac{27}{343}$	$\frac{108}{343}$	$\frac{144}{343}$	$\frac{64}{343}$

c

X	0	1	2	3
$P(X = x)$	$\frac{125}{216}$	$\frac{75}{216}$	$\frac{15}{216}$	$\frac{1}{216}$

3 **d**

X	2	3	4	5	6	7	8	9	10	11	12
$P(X = x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

e

X	0	1	2	3
$P(X = x)$	0.512	0.384	0.096	0.008

4 **a**

X	0	1	2	3	4	5
$P(X = x)$	0	$\frac{1}{55}$	$\frac{4}{55}$	$\frac{9}{55}$	$\frac{16}{55}$	$\frac{25}{55}$

b

X	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{21}$	$\frac{2}{21}$	$\frac{3}{21}$	$\frac{4}{21}$	$\frac{5}{21}$	$\frac{6}{21}$

c

X	7	8	9	10
$P(X = x)$	$\frac{6}{30}$	$\frac{7}{30}$	$\frac{8}{30}$	$\frac{9}{30}$

4 **d**

X	12	13	14	15
$P(X = x)$	$\frac{9}{42}$	$\frac{10}{42}$	$\frac{11}{42}$	$\frac{12}{42}$

5 **a** $k = \frac{1}{9}$

X	3	4	5
$P(X = x)$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{4}{9}$

b $k = \frac{1}{74}$

X	4	5	6
$P(X = x)$	$\frac{15}{74}$	$\frac{24}{74}$	$\frac{35}{74}$

5 **c** $k = \frac{1}{225}$

Y	1	2	3	4	5
$P(Y = y)$	$\frac{1}{167}$	$\frac{8}{167}$	$\frac{27}{167}$	$\frac{64}{167}$	$\frac{125}{167}$

d $k = 35$

B	1	2	3	4	5
$P(B = b)$	$\frac{5}{35}$	$\frac{6}{35}$	$\frac{7}{35}$	$\frac{8}{35}$	$\frac{9}{35}$

6 a

X	0	1	2	3
$P(X = x)$	$\frac{6}{504}$	$\frac{108}{504}$	$\frac{270}{504}$	$\frac{120}{504}$

b

 $\frac{249}{252}$

7 a

X	0	1	2	3
$P(X = x)$	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$

b

 $\frac{5}{32}$

8 a

Y	0	1	2	3	4
$P(Y = y)$	$\frac{1}{126}$	$\frac{20}{126}$	$\frac{60}{126}$	$\frac{40}{126}$	$\frac{5}{126}$

b

 $\frac{125}{126}$

Chapter 21

Exercise 2

- 1 a $b = 0.3$ $E(X) = 4.7$ b $b = 0.15$ $E(X) = 3.35$ c $b = 0.28$ $E(X) = 3.06$ d $b = 0.1$ $E(X) = 2.6$
- 2 $E(X) = \frac{1}{2}$ 3 a $c = \frac{12}{91}$ b $E(X) = \frac{441}{91}$ 4 a $E(X) = \frac{x - 3}{4}$ b $x = 3$
- 5

X	-1	1
$P(X = x)$	0.3	0.7

6

X	-1	1	3
$P(X = x)$	0.15	0.25	0.6

7

Y	0	2	4	6
$P(Y = y)$	$\frac{4}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{4}{10}$
- 8 $E(X) = \frac{25}{9}$ 9 $E(X) = 1$ 10 a 2.05 b 5.45 c 3.10 d 8.15 11 a 3 b 14 c 7 d 8
- 12 a $\frac{691}{400}$ b 5.8 c 6 d 0.986 13 a 7 b $\frac{329}{6}$ c $\frac{35}{6}$ 14 a $\frac{9}{7}$ b $\frac{15}{7}$ c $\frac{24}{49}$ 15 a $\frac{1}{14}$ b 2.53 c 1.65
- 16 $E(Y) = \frac{28}{15}$, $Var(Y) = 0.780$

Y	0	1	2	3	4
$P(Y = y)$	$\frac{2}{39}$	$\frac{56}{195}$	$\frac{84}{195}$	$\frac{8}{39}$	$\frac{1}{39}$

17 a $\frac{1}{2}$ b $\frac{26}{7}$ c 1.44
- 18 a can be a probability density function Mean = $\frac{110}{35}$ Standard deviation = 1.05
- 19 a 0, 1, 2, 3, 4 b

X	0	1	2	3	4
$P(X = x)$	$\frac{15}{24}$	$\frac{4}{24}$	$\frac{4}{24}$	0	$\frac{1}{24}$

c Mean = $\frac{2}{3}$ Variance = 1.06
- 20 a 10, 11, 12, 13, 14 b

X	10	11	12	13	14
$P(X = x)$	$\frac{6}{90}$	$\frac{24}{90}$	$\frac{30}{90}$	$\frac{24}{90}$	$\frac{6}{90}$

c $E(X) = 12$, $Var(X) = \frac{16}{15}$ d $\frac{25}{81}$
- 21 a $\frac{1}{14}$ b $E(X) = \frac{4}{7}$, $Var(X) = 0.816$ c $\frac{3}{196}$ d $E(Y) = \frac{8}{7}$, $Var(Y) = 1.63$

Chapter 21

Exercise 3

- 1 a 0.27 b 0.532 c 0.0556 2 a 0.201 b 0.833 c 0.834 3 a 0.208 b 0.0273 c 0.973 d 0.367 4 0.751 5 a 2.4 b 1.44 c 2
- 6 a 2.4 b 1.68 c 2 7 a $n = 7$, $p = \frac{1}{4}$, $q = \frac{3}{4}$ b 0.445 c 1 or 2 8 a 0.00345 b 0.982 c 0.939 9 a 0.0872 b 0.684
- 9 c 0.684 d 0.847 10 a 3.52×10^{-5} b 0.0284 c 0.683 d 0.163 11 a 0.238 b 0.0158 12 a 0.245 b 0.861 c 0.997
- 13 a 0.060 b 0.00257 c 0.998 d 0.24 e 0.978 14 a 10 b 2.74 c 0.416 15 a $\frac{4}{7}$ b 16 16 a 0.0258 b 0 c $\frac{1}{3}$ d 4
- 16 e 0.00258 f 0.00858 17 a Mean = 2.1 Variance = 1.81 b 2 c 0.204 d 0.148 e 0.042 18 a $X \sim Bin\left(8, \frac{1}{3}\right)$ b 0.156
- 18 c $\frac{8}{3}$ d 0.961 19 a 8 b 0.822 c 8 d 0.0108

Chapter 21

Exercise 4

- 1 a 0.244 b 0.423 c 0.353 d 3 2 a 0.134 b 0.151 c 0.554 d 6 3 a 0.125 b 0.332 c 0.933 d 10
- 4 a 1.68 b 0.0618 c 0.910 5 a 2.48 b 0.213 c 0.763 d 0.404 6 a 2.69 b 0.0799 c 0.136 d 0.505 e 0.944
- 7 a 2.10 b 0.0991 c 0.0204 d 0.350 8 a 7.62 b 0.996 9 a 1 b 0.981 10 a 0.905 b 0.00468 c 0.000151
- 11 a 0.874 b 0.191 c 0.223 d 0.0426 12 a 0.0804 b 0.751 c 2 d 0.119 e 0.173 13 a 0.0149 b 0.223
- 14 a 0.0183 b 0.215 c 0.975 d Mean = 80 Variance = 80 e 0.849 f 0.0262 15 a 0.0324 b 0.992 c 0.112 d 0.868 e 0.654
- 16 i a 0.345 b 0.753 c Mean = 42 Variance = 42 d 41 or 42 ii 0.111 17 a 0.195 b 0.785 c 0.152 d 0.166 e 8
- 18 a 0.0486 b 0.0499 c 0.00363 d 0.0000314

Chapter 21

Review Exercise

- 1 a $\frac{1}{10}$ b 3 c 1 2 a $\lambda = 2.99$ b 0.424 3 a 0.225 b 3 or 4 c 17.5 4 a 20 b 12.4 c 42.2 5 a 0.191 b 0.246
- 6 a 0.176 b 0.905 c 7 d 6.5 e 0.0158 7 a $\frac{12}{25}$ b $\frac{48}{25}$ 8 a 0.175 b 0.141 9 a 0.160 b 4 c 0.271 d 0.0808
- 10 a 0.0173 b 22 11 a i $\frac{1}{9}$ ii $\frac{1}{81}$ b i $\frac{73}{648}$ ii $\frac{575}{1296}$ c ii

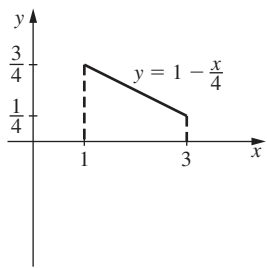
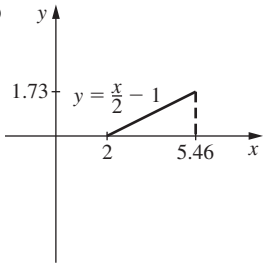
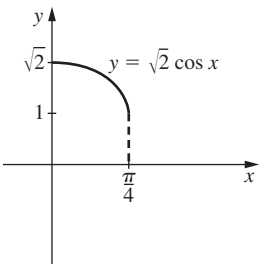
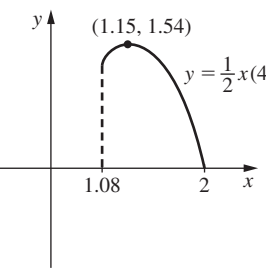
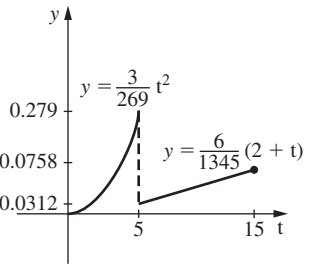
x	3	4	5
$P(X = x)$	$\frac{65}{1296}$	$\frac{175}{1296}$	$\frac{369}{1296}$

iii $\frac{6797}{1296}$
- 12 a 0.0105 b 0.0226 c 1 d 10 e 0.00116 13 30 14 a 0.222 b 0.939 c 0.104 d 0.00370 e 0.332 f 0.0145 g 0.995
- 15 a $P(A/B) = \frac{P(A \cap B)}{P(B)}$ b $P(A_1 \cup A_2) = P(A_1) + P(A_2)$ c $P(E_1 \cap E_2) = P(E_1) \times P(E_2)$
- 15 d This is a distribution that deals with events that either occur or do not occur, i.e. there are two complementary outcomes. We are usually told the number of times an event occurs and we are given the probability of the event happening or not happening.
- 15 e i ${}^4C_k \theta^k (1 - \theta)^{4-k}$ ii $E(X) = 4\theta$, $Var(X) = 4\theta(1 - \theta)$ iii $6\theta^2(1 - \theta)^2 + 4\theta^4(1 - \theta) + \theta^4$ iv 0.994 v $\frac{{}^4C_j \theta^j (1 - \theta)^{4-j}}{6\theta^2(1 - \theta)^2 + 4\theta^4(1 - \theta) + \theta^4}$
- 15 e vi 0.545, 0.0488 vii 0.969 16 a

Amount received in Euros X	30	25	15	12	18	25	40
$P(X = x)$	0.00137	0.0165	0.0823	0.248	0.329	0.263	0.0878
- 16 b Gain of 20.3 Euros c 59.5 d 305 17 $\alpha = \frac{1}{7}$, $E(X) = \frac{123}{49}$ 18 $P(X = r) = {}^nC_r \left(\frac{1}{2}\right)^n$ Mean = $\frac{n}{2}$ Standard deviation = $\sqrt{\frac{n}{4}}$ b i $\frac{5}{8}$ ii $\frac{8}{11}$

Chapter 22

Exercise 1

- 1 a $k = 1$ bc $\frac{1}{2}$ d $\frac{27}{32}$ 2 a $c = 5.46$ bc 0.0625 d 0.9975
- 3 a $k = \sqrt{2}$ bc 0.707 d 0.634 4 a $k = 1.08$ bc 0.306 d 0.621
- 5 a $\frac{3}{269}$ bc 0.494 d 0.375

Chapter 22

Exercise 2

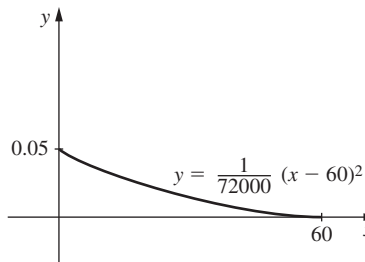
1 a $k = \frac{1}{2}$ b $\frac{4}{3}$ c $\frac{2}{9}$ 2 a $\frac{1}{\ln 3}$ b $\frac{2}{\ln 3}$ c $\frac{4}{\ln 3} - \frac{4}{(\ln 3)^2}$ 3 a $c = 0.755$ b 0.530 4 a $k = \frac{1}{16}$ b $\frac{23}{6}$ c 1.97 d 4 e $\frac{15}{32}$

5 a $k = \frac{1}{9}$ b 2.25 c $\frac{27}{80}$ d 2.38 6 a $c = \frac{1}{e}$ b 0.368 7 a $k = 1$ b 0.571 c 0.141 d $\frac{\pi}{6}$ e 0.830

8 a $k = 1.56$ b 0.616 c 0.178 d 0.546 e 0.384 9 a $-2 < x < 2$ b $0 < x < 0.421$ c 0.115 d 0.0209 10 a 1 b 0

10 c 0.273 12 a i 7.71 ii 0.947 b 0.318 13 a $k = \frac{e^2}{3(e^2 - 1)}$ b 0.148 c 0.00560 14 a

14 b $\sqrt{135}$ c 12.4 15 a $c = 0, k = \frac{3}{8}$ b 0.15 c $\frac{2}{\sqrt{3}}$ d 0.305



Chapter 22

Exercise 3

1 a 0.775 b 0.589 c 0.633 d 0.0392 e 0.9234 f 0.0973 g 0.203 h 0.562 i 0.841 j 0.5392 2 a 0.121 b 1.53
3 c -0.396 d -0.678 e 1.69 f -0.485 g 0.999 h 1.56 i 0.509 j 0.813 3 a 0.00332 b 0.901 c 0.00332 d 0.968
4 a 0.0912 b 0.997 c 0.952 d 0.122 e 0.125 5 a 0.106 b 0.809 c 0.998 d 0.101 e 0.0964 6 a 0.275 b 0.00139
6 c 0.683 d 0.0279 7 a 0.840 b 0.0678 c 0.683 d 0.997 8 a 40.6 b 38.9 c 41.9 d 39.2 9 a 93.9 b 84.6 c 86.6
9 d 82.2 10 a 5.89 b 13.7 c 18.1 d 4.18 11 Upper quartile $Z \geq 0.674$ Lower quartile $Z \leq -0.674$ 12 0.935 13 0.912 14 0.0443
15 0.939 16 0.134

Chapter 22

Exercise 4

1 15.1 2 75.6 3 30.5 4 39.0 5 6.81 6 7.81 7 14.1 8 11.2 9 $\mu = 11.6, \sigma = 4.53$ 10 $\mu = 46.3, \sigma = 4.26$

11 $\mu = 290, \sigma = 11.1$ 12 a $\mu = 23.6, \sigma = 6.13$ b 0.432 13 11.7

Chapter 22

Exercise 5

1 a 0.453 b 2.29 kg 2 a 10.6% b $m = 589g, n = 600g$ 3 a 0.309 b 0.227 c 0.440 4 a 0.106 b 0.734 c 0.599 d 0.159

4 e 0.606 f 0.292 5 a 440 b 82.3 kg 6 $\left(\mu - \sigma, \frac{1}{\sigma\sqrt{2e\pi}}\right), \left(\mu + \sigma, \frac{1}{\sigma\sqrt{2e\pi}}\right)$ 7 0.886 8 a 5 b 57.4 9 a 7.93 b 48.9

9 c 7 10 $\mu = 74.6, \sigma = 11.4$ 11 a 0.235 b 564g c 114 12 a 19.0 b 117 c 98.2 13 a 90.9% b 94.7 14 0.338

15 a $194 \leq X \leq 303$ b 0.423 16 a $\mu = 28.5, \sigma = \sqrt{1.49}$ b 0.587 17 a 126 b 280g 18 a 1.43 b 0.0146 19 4.14

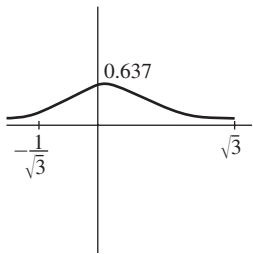
20 a 32.8 b 0.161 21 0.00123 22 $\mu = 59.3, \sigma = 18.6$ 23 a 4.82 b 0.0173

Chapter 22

Review Exercise

1 a $\mu = 34.5, \sigma = 3.93$ b 0.996 2 a 0.0668 b 142 cm c $q = 140 \text{ cm}, r = 180 \text{ cm}$ d 0.121 e 0.332 3 a 0.946 b 0.798

3 c 0.000109 d 0.99989 4 $2\frac{1}{2}$ 5 a 89.6% b 11.4 c 3.96% d 0.00110 6 a 0.0327 b 8.00 c Day 1: 2620. Day 2: 2610. 7 b 0



7 c 0.268 d 0.350 e 0.348 8 a $\frac{1}{4}$ b $E(X) = \pi, \text{Var}(X) = 2.93$ c 0.323

9 a 1.63 c 0.434 d \$6610 10 a $\mu = 28.6, \sigma = 14.3$ b 12.6% c $x = 0$ Model is not perfect. d i $\frac{8}{125}$

10 d ii $\frac{36}{125}$ iii $\frac{98}{125}$ 10 e $< 0.4^3$ f Either the events are not independent or the distribution is not continuous

11 a i $E(X) = \frac{1}{12} \int_0^2 x(8x - x^3) dx$ ii 1.24 b ii 1.29 c 1.63 12 a i 1.355 ii 110.37 b $A = 108.63, B = 112.11$ 13 a $\frac{4}{81}$ b 0.6

13 c 0.24 d 9000 cents. 14 0.783 15 b $e^{\frac{1}{4}} - e^{\frac{1}{4}} + \frac{1}{4}e$ c $E(X) = \frac{e}{2} - 1, \text{Var}(X) = 1 + \frac{e}{3} - \frac{e^2}{4}$ d 0.290 e 0.0243 f 0.179