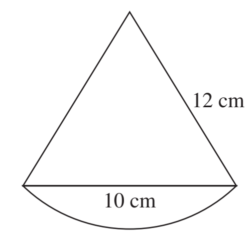
Revision Exercises

Trigonometry

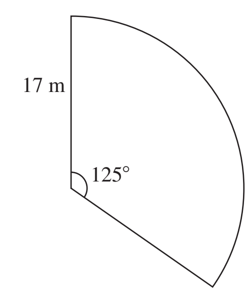
Revision Exercise 1

1 Express in radians: **a** 315° **b** 210°.

2 Find the area of this segment.

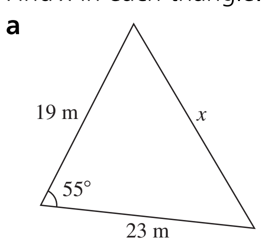


3 Find the length of this arc.



4 Find x in each triangle.

a



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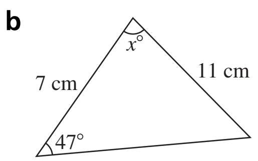

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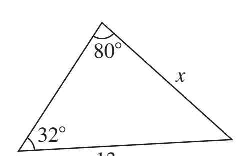

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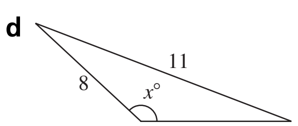


5 In the triangle ABC, $\angle A = 35°$, $BC = 5$ and $AB = 7$.
Find the two possible values of $\angle B$.

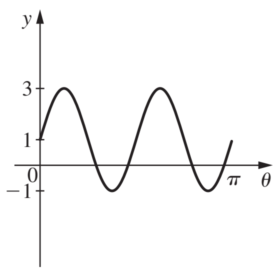
6 Sketch these graphs.

a $y = 4\sin(x + 30)°$ **b** $y = 7 - 2\cos 3\theta$

c $y = -3\csc\theta$ **d** $y = \arccos\theta$

7 State the equation of these graphs.

a



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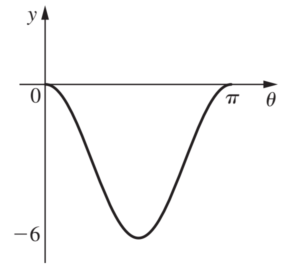


8 Solve these for $0 \leq \theta < \pi$.

a $2 \sin 3\theta + 1 = 0$ **b** $2 \sin \theta - \sqrt{3} = 0$

c $8 \tan 2\theta - 8 = 0$

9 Solve $4 \cos x = \tan x$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.

10 A sea tide is modelled by $y = 4 \sin\frac{\pi}{6}t$, in metres above mean sea level at time t hours after midnight.

a What is the height above mean sea level at 0400?

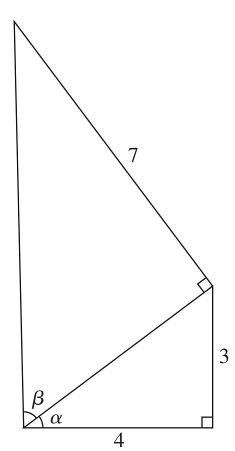
b When is the first time that the height above mean sea level is 2 metres?

c Sketch the graph of $y = 4 \sin\frac{\pi}{3}t$ for $0 \leq t \leq 12$.

11 Show that $\cos(A - B) - \cos(A + B) = 2 \sin A \sin B$.

12 Solve the equation $3 \sin \theta - 2 \cos 2\theta + 1 = 0$ for $0 \leq \theta < 2\pi$.

13 In the following diagram, find $\cos(\alpha + \beta)$, giving your answer in surd form with a rational denominator.



14 Express $\sqrt{2} \cos 2x° - 4 \sin 2x°$ in the form $k \cos(2x - \alpha)°$.
Hence solve $\sqrt{2} \cos 2x° - 4 \sin 2x° + 5 = 1$ for $0° \leq x° < 90°$.

15 Prove that $\cos 3\theta \equiv 4 \cos^3 \theta - 3 \cos \theta$.

16 Prove that $\dfrac{\sin 4\theta(1 - \cos 2\theta)}{\cos 2\theta(1 - \cos 4\theta)} = \tan \theta$ for $0 < \theta < \frac{\pi}{2}, \theta \neq \frac{\pi}{4}$.

Functions

Revision Exercise 2

1 Given that $y = 2x^2 - 9x - 5$ $(x \in \mathbb{R})$, find the set of values of x for which $y < 0$.

2 Find k if, for the equation $2kx^2 - 6kx + k + 7 = 0$, the roots are equal.

3 Solve these simultaneous equations:

$$y - x = 3$$
$$2x^2 + 5xy + y^2 = 38.$$

4 Knowing that the values of x satisfying the equation $3x^2 + kx + 2k = 0$ are real numbers, determine the range of possible values of $k \in \mathbb{R}$.

5 For $f(x) = x^2 + x - 4$, find:

a $f(4x)$ **b** $f(2x + 1)$ **c** $f\left(\frac{1}{x}\right)$.

6 For $f(x) = 7 - 2x$ and $g(x) = \frac{x + 1}{x - 1}, x \neq 1$, find:

a $f(g(x))$ **b** $g(f(x))$ **c** $f(f(x))$ **d** $g(g(x))$.

7 For $f(x) = x^2 + 5x - 24$, sketch the graph of $y = |f(x)|$ and $y = f(|x|)$.

8 Solve $|11 - 3x| = 7$.

9 Solve $\frac{2}{4x - 1} \leq 3$ for $x > 0$.

10 The one-to-one function f is defined on the domain $x > 0$ by $f(x) = \frac{2x + 1}{x + 3}$.

a State the range, A, of f.

b Obtain an expression for $f^{-1}(x)$, for $x \in A$.

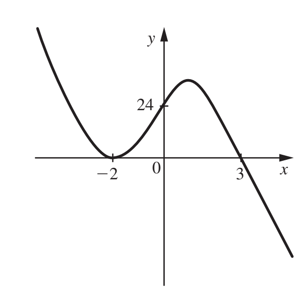
11 Solve the inequality $|x - 4| \geq |2x + 3|$.

12 Show that $x - 2$ is a factor of $2x^3 + x^2 - 7x - 6$ and hence find the other factors.

13 Factorise fully $f(x) = 2x^4 + 9x^3 + 2x^2 - 9x - 4$.

14 Show that $x = -3$ is a root of $x^3 + 4x^2 - 27x - 90 = 0$, and hence find the other roots.

15 Find an expression for $f(x)$ from its graph.



16 Using algebraic long division, find $(x^4 - 3x^2 + x - 4) \div (x - 1)$.

17 When the polynomial $f(x) = x^3 + 4x^2 + ax + b$ is divided by $x - 2$, the remainder is 18. $x + 1$ is a factor of $f(x)$. Find the values of a and b.

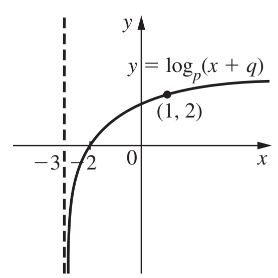
18 Simplify: **a** $\dfrac{x^7 \times x^2}{x^3}$ **b** $\dfrac{5p^{\frac{1}{2}} \times 3p^{\frac{3}{4}}}{p^{\frac{3}{2}}}$ **c** $x^{-\frac{1}{2}}\left(2x^{\frac{1}{2}} + 4x^{-\frac{3}{2}}\right)$.

19 Simplify: **a** $2 \log_a 5 + \log_a 3$ **b** $\log_p 12 + \log_p 6 - \log_p 8$
c $\log_6 8 + \log_6 3 - \log_6 4$.

20 Solve these for $x > 0$.
a $\log_a x + \log_a 6 = \log_a 54$ **b** $\frac{1}{2}\log_a x + \log_a 5 = \log_a 45$
c $\log_5(x - 1) + \log_5(x + 2) = \log_5 10$ **d** $\log_3(x + 2) - \log_3(x - 1) = 2$

21 Solve these for x.
a $\ln x = 12$ **b** $5^x = 7$ **c** $4e^x = 21$

22 The sketch shows part of the graph of $y = \log_p(x + q)$.
What are the values of p and q?



23 Find the exact value of x satisfying this equation:
$(8^x)(3^{2x+1}) = 6^{x+2}$.

24 Solve the simultaneous equations $\log_x y = 1$ and $xy = 36$ for $x, y > 0$.

Complex Numbers, Binomial Theorem, Sequences and Induction

Revision Exercise 3

1 Find the modulus and argument of the complex number $\dfrac{3 + 4i}{i + 2}$.

2 Find all of the roots, real and imaginary, of the equation $z^3 - 10z^2 + 37z - 52 = 0$.

3 Prove by induction that for $t > 0$, $(1 + t)^n \geq 1 + nt$ for all positive integers n.

4 Given that $|z + 2| = |z - i|$, where $z = x + iy$, show that $ax + by + c = 0$, thus finding $a, b, c \in \mathbb{Z}$.

5 Find the coefficient of x^5 in the binomial expansion of $\left(3 - \frac{1}{2}x\right)^7$.

6 A geometric sequence has all positive terms. The sum of the first two terms is 160 and the sum to infinity is 180. Find the value of
a the common ratio
b the first term.

7 Find the sum of the positive terms of the sequence 101, 92, 83,…

8 **a** Expand $(\cos\theta + i\sin\theta)^5$ using the binomial theorem.
b Expand $(\cos\theta + i\sin\theta)^5$ using de Moivre's theorem.
c By equating imaginary parts, express $\sin 5\theta$ in terms of $\sin\theta$.

9 Find the fifth roots of unity and illustrate these on an Argand diagram.

10 What is the coefficient of x^7 in the expansion of $(2x + 1)^4\left(3x - \frac{1}{x}\right)^6$?

11 Using mathematical induction, prove that $n^7 - n$ is divisible by 42.

12 **a** $x + 1, 3x + 1, 6x - 1$ are the first three terms of an arithmetic sequence. For what value of n does S_n, the sum of the first n terms, first exceed 100?
b The sum of the first three terms of a positive geometric sequence is 52 and the sum of the 5th, 6th and 7th terms is 4212. Identify the first term and the common ratio.
c For what value of n does this geometric series have a sum of 354 292?

13 Prove, using mathematical induction, that for all positive integers n,
$[r(\cos\theta + i\sin\theta)]^n = r^n[\cos n\theta + i\sin n\theta]$ where $i^2 = -1$.

14 Prove, using mathematical induction, that for $A = \begin{pmatrix} 1 & 0 \\ 0 & p \end{pmatrix}$, $A^n = \begin{pmatrix} 1 & 0 \\ 0 & p^n \end{pmatrix}$ for all positive integers n.

15 Find a cubic equation with real coefficients, given that two of its roots are -4 and $2 + i\sqrt{2}$.

16 **a** Use the binomial theorem to expand $(3 - x)^4$.
b Express $z = \sqrt{3} + i$ in $re^{i\theta}$ form.
c Use de Moivre's theorem to find z^6.
d Using mathematical induction, prove that $\dfrac{d^n}{dx^n}(e^{3p^2x}) = 6^n p^n e^{px}$, $\forall\, n \in \mathbb{Z}^+$.

Calculus

Revision Exercise 4

1 Differentiate these functions with respect to x.
a $y = x^3 \sin 2x$
b $y = \ln(x + 1)\tan 3x$
c $f(x) = \dfrac{e^{2x}}{x^3 - 6}$
d $y = \dfrac{6(3\sin x - 2)^3}{(5x - 12)^2}$
e $4x^{\frac{1}{2}} + y^3 = 25$
f $y = x\cos^{-1}\sqrt{x + 4}$
g $y = (x + \ln 3x)^4$
h $y = e^{2\sin^2 x}$

2 Evaluate the following indefinite integrals.

a $\int (3x^{\frac{1}{3}} - 2x^{\frac{3}{4}})^2 \, dx$

b $\int x^2 e^{-5x} \, dx$

c $\int \frac{6}{\sqrt{36 - 25x^2}} \, dx$

d $\int \frac{3x}{(4 + 3x)^4} \, dx$

e $\int x^3 \ln 5x \, dx$

f $\int e^{-2x} \sin 4x \, dx$

g $\int \cos^{-1} 3x \, dx$

h $\int \sqrt{16 - x^2} \, dx$ using the substitution $x = 4 \sin \theta$

3 Find the equation of the tangent to the curve $y = \sqrt{x^2 - 5}$ at the point (3, 2).

4 The region R is bounded by the x-axis and the part of the curve $y = \sin 3x$ between $x = 0$ and $x = \frac{\pi}{3}$. Find

a the area of R

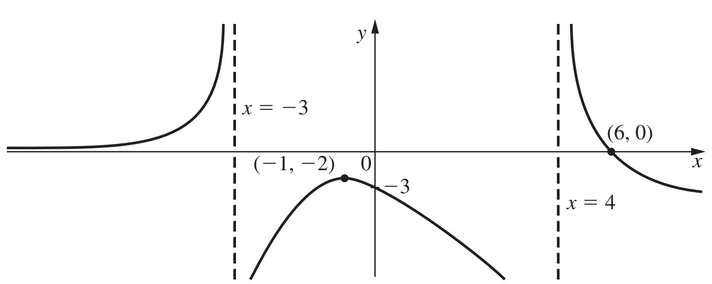
b the volume of the solid formed when R is rotated through 2π radians about the x-axis.

5 The equation of a curve is $(x + y)^2 + 3(x - y)^2 = 36$.

a Find a formula for the gradient of any point on the curve in terms of x and y.

b Find the coordinates of the points where the tangent to the curve is parallel to the x-axis and the coordinates of the points where the tangent to the curve is parallel to the y-axis.

6 Consider the graph of $y = f(x)$.



a Draw the graph of the derivative of $y = f(x)$.

b Draw the graph of $y = \frac{1}{f(x)}$.

7 Differentiate $f(x) = 2x^3 - 3x + 4$ from first principles.

8 Consider the curve $y = x^3 - 2x^2 - 4x + 5$.

a Find the stationary points on the curve.

b Determine whether these are maximum points or minimum points.

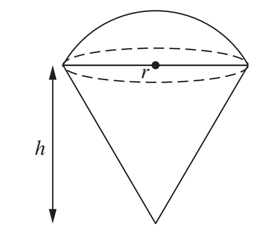
c Find the point of inflexion on the curve.

d Determine the range of values for which the curve is concave up and concave down.

9 If $y = e^x \ln(\sin 2x)$ find $\frac{d^2y}{dx^2}$ at the point where $x = \frac{\pi}{8}$.

10 Find the coordinates of any stationary points on the curve $y = \frac{e^x}{x - 1}$ and determine their nature.

11 A container is modelled using a cone with a hemisphere attached at one end as shown in the diagram below.



a Given that the height of the cone is h and the radius of the hemisphere is r, find the volume of the solid formed.

b If the volume is 10π cm^3 find a formula for the surface area of the container in terms of r only.

c Find the value of r for which the surface area is a minimum.

12 Evaluate $\int_2^3 x\sqrt{3x - 4} \, dx$.

13 Find the area bounded by the curve $y = 2x^2 e^{-x}$, the x-axis and the lines $x = 0$ and $x = 2$.

14 Consider the curve $y = \frac{6}{9 + x^2}$.

a Find the coordinates of any maximum or minimum points.

b Find the equations of any asymptotes.

c Sketch the curve.

d Find the area bounded by the curve, the y-axis, the line $x = 5$ and the line $y = 1$.

15 Show that the solution to the differential equation $y\frac{dy}{dx} = x(1 + y^2)$, given that $y = 1$ when $x = 0$, is of the form $y^2 = Ae^{x^2} - B$. Find the values of A and B.

16 A particle moves in a straight line with velocity, in metres per second, at time t seconds, given by $v(t) = 4t^3 - 9t + 6, t \geq 0$.

a Calculate the total distance travelled by the particle in the first three seconds of motion.

b Calculate the minimum velocity of the particle.

c Calculate the acceleration of the particle after 6 seconds.

17 The region A is bounded by the curve $y = \tan\left(\frac{1}{2}x + \frac{3\pi}{16}\right)$ and by the lines $x = 0$ and $x = \frac{\pi}{8}$. Find the exact value of the volume formed when the region A is rotated fully about the x-axis.

18 Find the solution to the differential equation $\frac{d\theta}{dt} = \theta e^{-t} \sin 2t$ given that $t = 0$ when $\theta = \frac{\pi}{2}$.

19 Oil drips from a can onto the floor, forming a thin circular film on the ground. The rate of increase of the radius of the circular film is given by the formula $\frac{dr}{dt} = \frac{2}{1 + t^2}$. Given that when $t = 1$ sec, the radius of the film is 3 cm find a general formula for the radius r at any time t.

20 The surface area, A, of a sphere is increasing at a rate of $5\ \text{cm}^2\text{s}^{-1}$. Find the rate of increase of the volume, V, of the sphere when the radius of the sphere is 4 cm.

21 A curve has equation $\frac{x^2}{16} - \frac{y^2}{25} = 9$.

a Find the gradient of any point on the curve.
b Find the equations of all the asymptotes.

Matrices and Vectors

Revision Exercise 5

1 Find the value of p for which the following system of equations does not have a unique solution.

$3x - 2y + 2z = 1$
$x + 3y = -8$
$5x - 7y + pz = 10$

2 Find the angle between the vectors $\mathbf{a} = 3\mathbf{i} + \mathbf{j} - 5\mathbf{k}$ and $\mathbf{b} = 5\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$. Give the answer in radians.

3 The equations of two planes π_1 and π_2 are

π_1: $4x + 5y - z = 15$;
π_2: $\mathbf{r} = 2\mathbf{i} + \lambda(\mathbf{i} + 2\mathbf{j}) + \mu(3\mathbf{i} - 2\mathbf{k})$.

a Write the equation of plane π_2 in the form $\mathbf{r}\cdot\hat{\mathbf{n}} = d$.
b Find the angle between the two planes.
c Find the line of intersection of the two planes.

4 If $M = \begin{pmatrix} x & 7 \\ 3 & 5 \end{pmatrix}$ and $N = \begin{pmatrix} 1 & y \\ 6 & 4 \end{pmatrix}$, find the values of x and y, given that the matrix product MN is commutative.

5 Given that $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$, $\mathbf{b} = -\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $\mathbf{c} = -4\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, find the value of $3(\mathbf{a} \times \mathbf{b})\cdot\mathbf{c}$.

6 The position vectors of the points A and B are given by $\overrightarrow{OA} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $\overrightarrow{OB} = \mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$ where O is the origin. Find a vector equation of the straight line passing through A and B. Given that this line is perpendicular to the vector $q\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$, find the value of q.

7 The variables x, y and z satisfy the simultaneous equations

$2x - y + 3z = 4$
$3x - 5y + 4z = 9$
$-5x + 13y - 6z = p$

where p is a constant.

a Show that these equations do not have a unique solution.
b Find the value of p which makes these equations consistent.
c For this value of p, find the general solution to these equations.

8 The vectors **a, b, c** and **d** are defined as:

$\mathbf{a} = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 5 \\ -3 \\ 2 \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} 4 \\ -1 \\ p \end{pmatrix}$ and $\mathbf{d} = \begin{pmatrix} 3 \\ q \\ -5 \end{pmatrix}$.

The vector $\mathbf{s}$ is defined as $(\mathbf{a}\cdot\mathbf{b})\mathbf{c} + (\mathbf{b} \times \mathbf{d})$. Find the connection between p and q if $\mathbf{s}$ is perpendicular to $\mathbf{a}$.

9 Determine whether the lines $\mathbf{r} = \begin{pmatrix} 3 \\ 6 \\ 5 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ -3 \\ 7 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 8 \\ -3 \\ 9 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$ intersect.

If so, find the position vector of the point of intersection.

10 Find the values of x such that $\begin{vmatrix} x & 3x \\ -x & 4 \end{vmatrix} = \begin{vmatrix} -1 & 3 & 2 \\ 4 & x & 0 \\ -3 & 6 & 2 \end{vmatrix}$.

11 Given $\mathbf{a} = 2\mathbf{i} - 5\mathbf{j} + 6\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} + p\mathbf{j} + \mathbf{k}$, and that the vector $(3\mathbf{a} - 4\mathbf{b})$ has magnitude $\sqrt{353}$, find the possible values of p.

12 **a** Find the line of intersection of the planes $\mathbf{r}\cdot\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = 3$ and $\mathbf{r}\cdot\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = 1$.

b Show that this line is contained in the plane $\mathbf{r}\cdot\begin{pmatrix} 3 \\ -4 \\ 5 \end{pmatrix} = -1$.

c Verify using row reduction that the planes

$x + 2y + z = 3$
$2x - y + 3z = 1$
$3x - 4y + 5z = -1$

all meet in a common line.

13 **a** Find the inverse of the matrix $A = \begin{pmatrix} k - 1 & -1 \\ 2 & k \end{pmatrix}$ where $k \in \mathbb{R}$.

b Hence solve these simultaneous equations.

$kx - y - x = 4k$
$2x + ky = 2 - k^2$

14 The points P, Q, R, S have position vectors **p, q, r, s** given by

$\mathbf{p} = 3\mathbf{i} + 2\mathbf{k}$
$\mathbf{q} = -\mathbf{j} + 5\mathbf{k}$
$\mathbf{r} = -3\mathbf{i} - 6\mathbf{j} + 7\mathbf{k}$
$\mathbf{s} = -2\mathbf{j} - 3\mathbf{k}$

respectively. Find

a the vector equation of the line PQ
b the area of the triangle PQR
c the area of the parallelogram $PQRS$

d the equation of the plane containing P, Q and R

e the angle between the line PS and the plane containing P, Q and R

f the perpendicular distance of S from the plane containing P, Q and R.

15 The point $P(3, 4, -2)$ is on the line L, which is perpendicular to the plane π with equation $2x - y + 3z - 24 = 0$.

a Find the Cartesian equation of the line L.

b Find the coordinates of the point of intersection of the line L and the plane π.

c The point P is reflected in the plane π to the point P'. Find the coordinates of P'.

d Calculate the distance PP'.

e Find the equation of the line in parametric form which passes through the points P' and Q which has coordinates $(3, -2, 3)$.

Statistics and Probability

Revision Exercise 6

All questions in this exercise require a calculator.

1 A discrete random variable X has the following probability distribution.

x	0	1	2	3	4
$P(X = x)$	0.15	0.22	0.38	0.2	0.05

Find:

a $E(X)$

b $E(X^2)$

c $E(2X - 1)$

d $Var(X)$.

2 The marks x from an IB mathematics examination are shown below. The examination is marked out of 120.

78	65	92	105	37	44	81	70
34	95	76	40	99	59	39	64
110	61	37	91	84	51	27	12
87	78	69	57	37	61	68	30

a Draw a box and whisker plot of this data.

b Calculate the interquartile range.

c Find the mean mark.

d Construct a cumulative frequency table using $x \le 10$, $x \le 20$, $x \le 30 \ldots$

e Draw a cumulative frequency diagram of this information.

f Estimate the values of the median and the interquartile range from the cumulative frequency diagram.

3 A continuous random variable X has a probability density function

$$f(x) = \begin{cases} kx^3 & \text{for } 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

Find:

a the value of k

b $P(1 \le X \le 1.5)$

c $E(X)$

d $Var(X)$

e the median of X.

4 How many different arrangements, each consisting of four different digits, can be formed from the digits 1, 2, 3, 4, 5, 6, 7 if

a **i** the last number of the arrangement is odd?

ii three of the four digits are odd?

iii the arrangement must contain the digit 3?

b What is the probability of an arrangement chosen at random starting with an odd number and ending with an even number?

5 A manufacturer of nails finds that the boxes they produce for sale contain nails whose lengths are normally distributed with mean 2.5 cm and standard deviation 0.75 cm.

a What is the probability that a nail picked at random has a length greater than 2.7 cm?

b What is the probability that a nail picked at random has a length less than 2.1 cm?

c What is the probability that a nail picked at random has a length between 2 cm and 3 cm?

d If there are 250 nails in a box, what is the expected number with a length of more than 3.1 cm?

e What is the maximum length of any nail in the bottom 25% of the distribution?

f Given that a box is rejected if more than 4% of nails are less than a certain length, what length is this?

6 In a factory making packets of biscuits a sample of 100 packets is taken. The mass of each packet is shown in the table below.

Mass (g)	**Frequency**
245	2
246	5
247	12
248	18
249	21
250	19
251	13
252	10

a Calculate the median and the interquartile range.

b Calculate the mean of the sample.

c Calculate the standard deviation of the sample.

d Calculate an unbiased estimate for the standard deviation of the population.

It is decided to model the distribution using a normal distribution X with the mean calculated above and the standard deviation for the population.

e Find $P(X \ge 250)$.

f Given that 50% of the packets of biscuits have masses between m grams and n grams, where m and n are symmetrical about 248 grams and $m < n$, find the values of m and n correct to 1 decimal place.

g What mass do more than 87% of packets of biscuits have? Give the answer correct to 1 decimal place.

7 If $X \sim Po(m)$ and $E(X^2) = 2.5$, find:

a m

b $P(X = 4)$

c $P(X \ge 3)$

8 Each morning, Pierre decides whether to cycle to school or to take the bus. The probability he cycles is 0.4. If he cycles to school the probability that it rains is 0.65 and if he takes the bus to school the probability that it rains 0.25.

a What is the probability that he takes the bus to school and it does not rain?

b What is the probability that it rains on the way to school?

c Given that it rains, use Bayes' theorem to determine the probability that he took the bus to school.

9 Jean is practising his tennis serve. The number of times he hits the service box follows a binomial distribution. The expectation that he hits his service in the service box is 4.08 and the variance is 1.3056. Find

a the number of times he practises his serve

b the probability that he hits the serve in the service box

c the probability that he hits the serve in the service box three times

d the probability that hits the serve in the service box more than four times

e the probability that he misses the service box at least once.

10 Bill is trying to buy a new computer and decides to buy it from one of three companies. The probability that he buys it from company A is 0.4, the probability that he buys it from company B is 0.45 and the probability he buys it from company C is 0.15. If he buys the computer from company A, the probability it will develop a fault within the first year is 0.05, if he buys from company B it is 0.09 and if he buys from company C it is 0.11.

a Calculate the probability that Bill buys a computer from either company A or company B and that it does not develop a fault within the first year.

b Calculate the probability that the computer that Bill buys will develop a fault within the first year.

c Given that Bill's computer develops a fault, what is the probability that he bought it from company C?

11 A dressmaker is making dresses on a machine and occasionally minor faults are produced. The number of dresses with minor faults follows a Poisson distribution with mean 0.9. Calculate

a the probability of exactly two dresses with minor faults

b the probability of at least two dresses with minor faults

c the most likely number of dresses with minor faults

d the probability of at least one dress with a minor fault.

12 Bella plays a game with two fair cubical dice. If the sum of the scores is a prime number she gains \$3, if the sum of the scores is an even number other than 2 she gains \$1. Otherwise she loses \$5. Let X be the discrete random variable "Bella's gain".

a Write down the probability distribution for X.

b Calculate $E(X)$.

c Calculate $Var(X)$.

d How much should Bella gain or lose when the sum of the scores is a prime number in order that the game is fair?

13 The probability of a car battery failing x months after manufacture is given by a continuous random variable with probability density function

$$f(x) = \begin{cases} \frac{k}{12}x^2 & 0 \le x \le 24 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

a Find the value of k.

b Find the probability that a car battery fails within 9 months of manufacture.

c Find the probability that it fails more than one year after manufacture.

d A certain vehicle needs four batteries and all need to be working for the engine to start. Find the probability that the engine will start more than one year after the batteries have been manufactured.

14 Charlie's Cake Company's best selling cake is their Gateau Supreme. The mass of each cake is normally distributed with a mean of 300 grams and a standard deviation of 30 grams. A cake is rejected if the mass is less than 265 grams or more than 325 grams.

a Find the percentage of cakes that are accepted.

The settings on the machines which make the Gateau Supreme are altered so that the mean mass changes but the standard deviation remains unchanged. With the new settings 12% of cakes are rejected because the mass is too high.

b Find the new mean mass of the Gateau Supreme, giving your answer to 3 significant figures.

c Find the percentage of cakes which are now rejected because the mass is too low.

d Five Gateau Supremes are chosen at random. What is the probability that exactly two of them will be rejected because the mass is too low?

15 The probability of finding a mistake on a particular page of a book is 0.09.

a In the first five pages of a book, what is the probability of finding exactly three mistakes?

b In the first eight pages of the book, what is the probability that there are at least two mistakes?

c What is the most likely number of mistakes in a chapter of 15 pages?

d What would be the expected number of mistakes in a 400 page book?

e Given that the first page has no mistakes, what is the probability that there are more than two mistakes in the next ten pages?